ECE329: Final Exam Review Session

March 12th, 2025

Part 1: Maxwell’s Equations

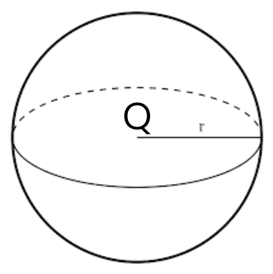
- Vector Calculus
- Maxwell’s Equations
- Electrostatics
- Magnetostatics
- Electrodynamics
- Capacitance, Conductance, Resistance, Inductance
- Polarization and Magnetization
- Boundary Conditions

Maxwell's Equations

Everywhere and always

Faraday's Law	$\oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\iint_S \vec{B} \bullet d\vec{S}$	$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$
Ampere's Law	$\oint_C \vec{H} \bullet d\vec{l} = \iint_S \vec{J} \bullet d\vec{S} + \frac{d}{dt}\iint_S \vec{D} \bullet d\vec{S}$	$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$
Gauss' Law	$\oiint_S \vec{B} \bullet d\vec{S} = 0$	$\nabla \bullet \vec{B} = 0$
Gauss' Law	$\oiint_S \vec{D} \bullet d\vec{S} = \iiint_V \rho dV$	$\nabla \bullet \vec{D} = \rho$

Gauss's Laws



For closed surfaces only:
- If there is more (electric OR magnetic) flux,
- Then there is more (electric or magnetic) field flowing out of the surface,
- Then, more free charge must be enclosed in the surface.

This is **Gauss's Law**!

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

Electrostatics & Magnetostatics

Electrostatic potential:

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

Magnetostatics (Ampere's Law):

$$I_{\mathrm{encl}} = \oint_C \vec{H} \cdot d\vec{l}$$

General Tips

To solve for fields:

1. Handle each unique volume separately.
2. Handle each direction of the field separately.
3. For static currents, magnetic fields cannot point in the same direction as currents. This eliminates one of the three directions!
4. Try using symmetry whenever possible to set things to zero.

It's usually obvious which direction you should be solving for…

General Tips

Using Gauss's Law or Ampere's Law to solve for fields:

1. Gauss's Law works with surfaces. Choose a surface such that your direction punches **through** it.

    Works well with Cartesian-like or radial directions.

2. Ampere's Law works with loops. Choose a loop such that your direction travels **along** that loop and current punches **through** it.

Electrodynamics

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_{S}\vec{B}\cdot d\vec{S} = \oint_{C}\vec{E}\cdot d\vec{l} = \varepsilon = IR$$

Ψ is **magnetic flux**. Units: [Wb]

In electrodynamics, $\vec{E}$ is no longer curl-free/path-independent/conservative; integration over a closed loop yields a nonzero number.
ε is **electromotive force** (or emf). Units: [V]

Electromotive ‘force’

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{l} = \varepsilon$$

ε is **electromotive force** (or emf). Units: [V]
It is not a force.

It is the electromagnetic work done to move a unit electric charge once around the closed loop.

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\varepsilon = IR$$

Capacitance

Capacitance: the ability of something to collect and store energy in the form of electrical charge.

This energy is stored as opposite electric charges being held apart.

$$Q = CV \qquad G = \frac{\sigma}{\epsilon} C \qquad R = \frac{1}{G}$$

Inductance

Inductance: the tendency of an electrical conductor to oppose a change in electric current flowing through it.

Inductance exists only in conductors!

For a conductor A:

- **Self inductance**: change in current flowing through A induces an emf in A itself.
- **Mutual inductance**: change in current flowing through a conductor nearby induces an emf in A.

$$\Psi = LI$$

P fields: The math

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: The defining equation.

Assumption: Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

- $\vec{P} = \epsilon_0 \chi_e \vec{E}$ with **electric susceptibility** $\chi_e \geq 0$ nearly always in this class.
- Let **electric permittivity** be $\epsilon = \epsilon_0(1 + \chi_e)$.
- Let **relative electric permittivity** be $\epsilon_r = 1 + \chi_e$
- $\vec{D} = \epsilon \vec{E}$

Bound charges:

- $\rho_b = -\nabla \cdot \vec{P}$

M fields: The math

$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$: The defining equation.
Assumption: Material is ‘isotropic’, so $\vec{M}$ is collinear to $\vec{H}$. Then:

- $\vec{M} = \chi_m \vec{H}$ with **magnetic susceptibility** $\chi_m \geq 0$ nearly always in this class.
- Let **magnetic permittivity** be $\mu = \mu_0(1 + \chi_m)$.
- Let **relative magnetic permittivity** be $\mu_r = 1 + \chi_m$
- $\vec{B} = \mu \vec{H}$

Bound current:

- $\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$

Boundary Conditions

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

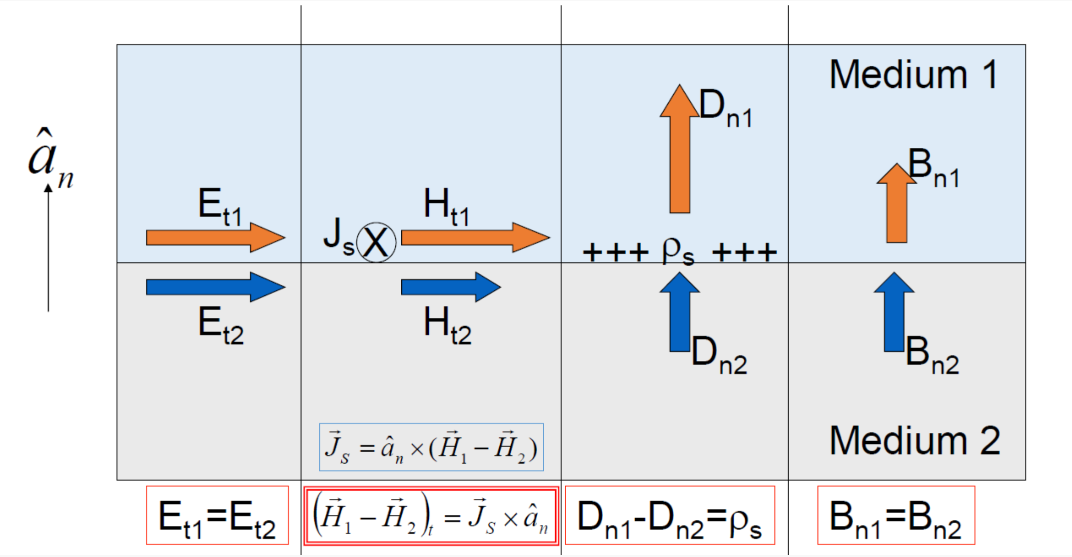
$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$



Part 2: Electromagnetic Wave Theory

- Phasors
- TEM Wave Propagation in Material Media
- Poynting Theorem
- Polarization
- Standing Waves

Phasor Notation

$$\vec{E}(x,t) = A\cos(\omega t - \beta x)\hat{z} = \mathrm{Re}\left\{Ae^{j\omega t}e^{-j\beta x}\hat{z}\right\} \leftrightarrow Ae^{-j\beta x}\hat{z} = \tilde{E}(x)$$

Time domain

Phasor

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

Wave Equation in Material Media

Assumptions: $\rho = 0$, $\vec{J} = 0$, i.e. region is a source-free. $\sigma \neq 0$ now!

Wave equation: $\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$. Solutions are TEM waves. $\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

General form of cosinusoidal solution:

EXACT Formulae

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

APPROXIMATE Formulae

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	45°	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

Wave Polarization

Polarization is how the tip of $\vec{E}$ varies over time.

Linear: In phase or 180 degrees out-of-phase
Circular: 90 degrees out-of-phase with equal magnitude
- Right-Handed
- Left-Handed

Elliptical: Anything else

Wave Reflection & Transmission

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_r(x) = E_0 \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{y}$$

$$\tilde{H}_r(x) = -\frac{E_0}{\eta_1} \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{z}$$

$$\tilde{E}_t(x) = E_0 \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{y}$$

$$\tilde{H}_t(x) = \frac{E_0}{\eta_2} \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Coefficients

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

What if $\eta_2 = 0$?

Poynting Vector & Theorem

$\vec{S} = \vec{E} \times \vec{H}$. Units: W/m². "Instantaneous" power per unit area passing through surface in direction of $\vec{S}$.

Energy unit volume balance equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E} + \frac{1}{2}\mu\vec{H}\cdot\vec{H}\right) + \nabla\cdot\vec{S} + \vec{J}\cdot\vec{E} = 0$$

Time-average:

$$\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \widetilde{H}^{*}\}$$

Part 3: Time-Domain Transmission Lines

- Bounce diagrams
- Steady-state
- Multiline circuits

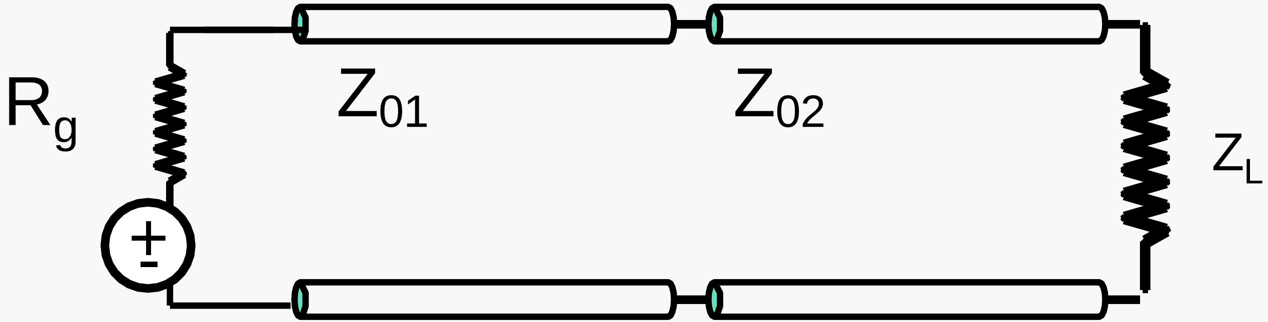
Basic TL: Time Domain, Not Steady State

Generator injection coefficient: $\tau_g = \frac{Z_0}{R_g + Z_0} \neq 1 + \Gamma_g$

Load reflection coefficient: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Generator reflection coefficient: $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$

Multiline TL Circuits (Time Domain)

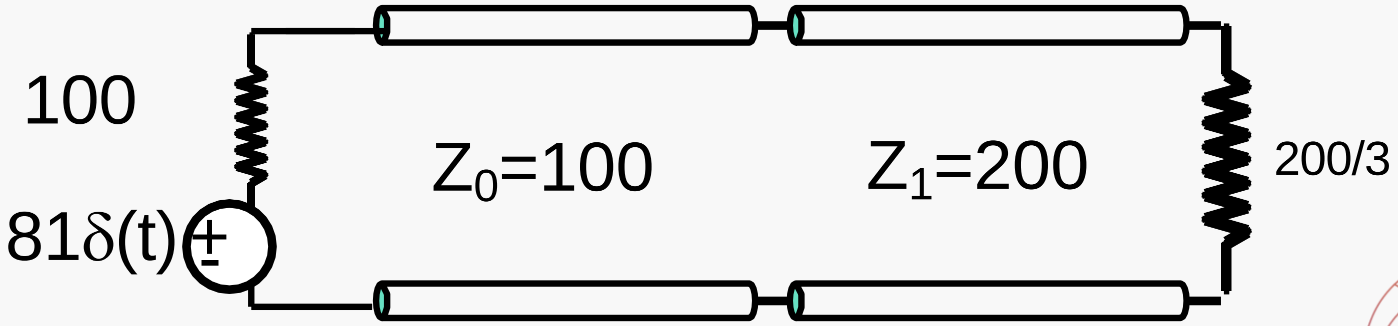


When we travel FROM line j TO line k:

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

Problem: Bounce Diagrams w/ Multiline



Input: $V_g = 81\delta(t)$ [V]

$Z_L = \frac{200}{3}\Omega, R_g = 100\Omega$

$Z_0 = 100\Omega, Z_1 = 200\Omega$

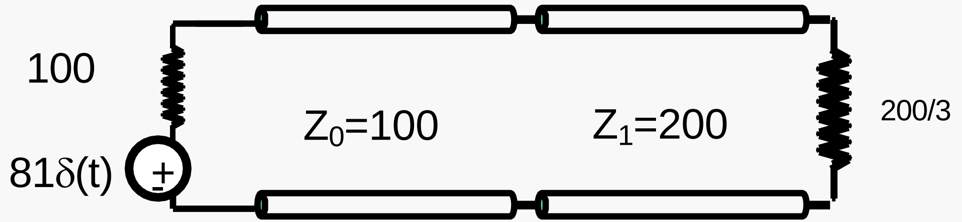
$v = c$

First TL length = 3 [m]. Second TL length = 4.5 [m].

Create a voltage bounce diagram for the first 45 nanoseconds.

Problem: Bounce Diagrams w/ Multiline

1. $\tau_g = \frac{Z_0}{R_g+Z_0} = \frac{1}{2}$
2. $V^+ = Vs * \tau_g = 81\delta * \frac{1}{2} = 40.5\ \delta$
3. $\Gamma_g = \frac{R_g-Z_0}{R_g+Z_0} = 0$
4. $\Gamma_L = \frac{R_L-Z_1}{R_L+Z_1} = -\frac{1}{2}$
5. $\Gamma_{01} = \frac{Z_1-Z_0}{Z_1+Z_0} = \frac{1}{3}$
6. $\Gamma_{10} = \frac{Z_0-Z_1}{Z_1+Z_0} = -\frac{1}{3}$
7. $\tau_{01} = 1 + \Gamma_{01} = \frac{4}{3}$
8. $\tau_{10} = 1 + \Gamma_{10} = \frac{2}{3}$



Remember:

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

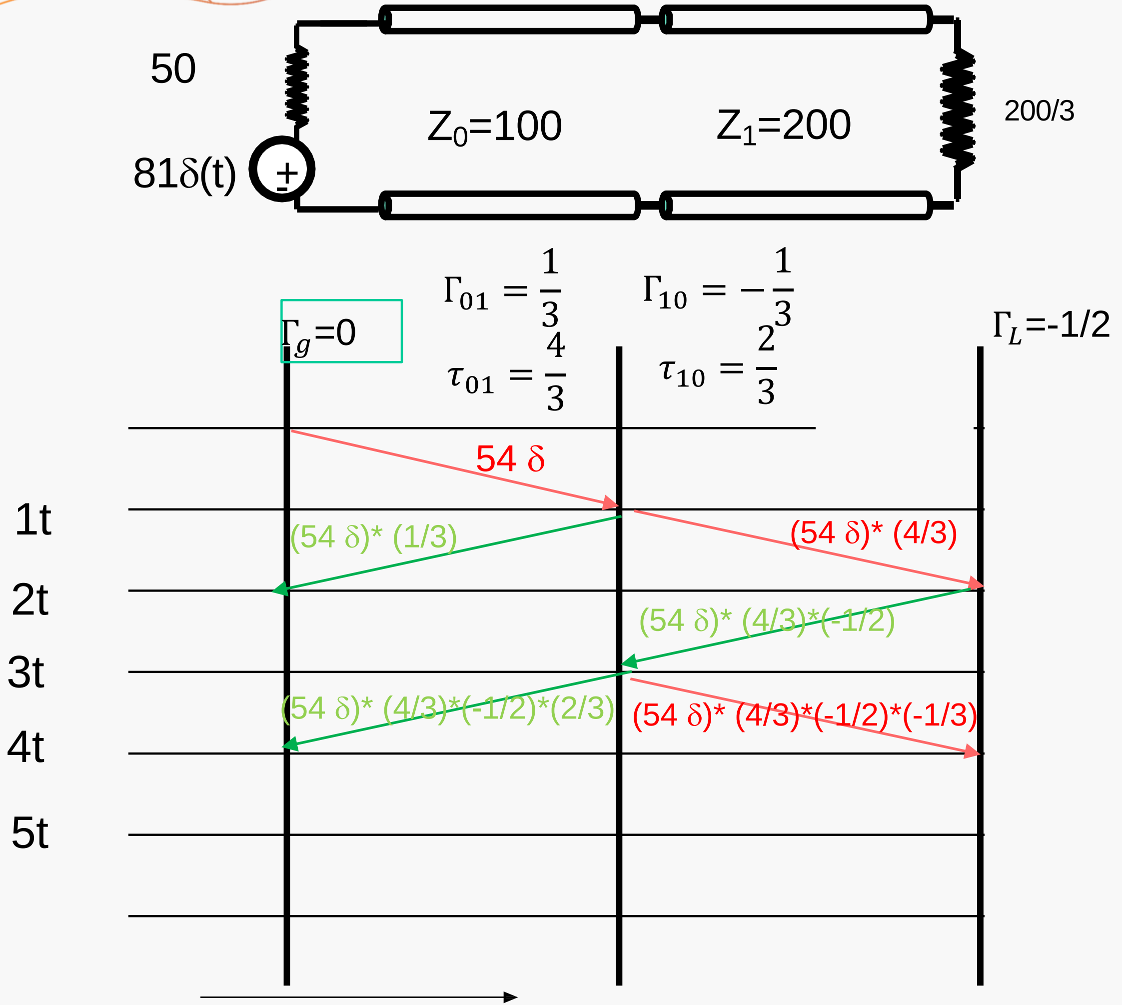
$$\tau_{jk} = 1 + \Gamma_{jk}$$

$$L_1 = 3\ m$$

$$L_2 = 4.5\ m$$

$$t_1 = \frac{L_1}{u} = 10\ ns$$

$$t_2 = \frac{L_2}{u} = 15\ ns$$


50
81δ(t)
+
-
Z0=100
Z1=200
200/3
Γ01 = 1/3
τ01 = 4/3
Γ10 = −1/3
τ10 = 2/3
Γg=0
ΓL=-1/2
54 δ
1t
2t
3t
4t
5t
(54 δ)* (1/3)
(54 δ)* (4/3)
(54 δ)* (4/3)*(-1/2)
(54 δ)* (4/3)*(-1/2)*(2/3)
(54 δ)* (4/3)*(-1/2)*(-1/3)

Multiline TL Circuits (Time-Domain)

Parallel resistor:

$$\Gamma_{12} = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0}$$

Z_eq = $\frac{ZZ_1}{Z+Z_1}$

$$\tau = 1 + \Gamma = 1 + \Gamma_{eq}$$

Multiline TL Circuits (Time-Domain)

Series resistor:

$$\Gamma_{12} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1}$$

$$\tau_{12} = \frac{2Z_2}{Z_{eq} + Z_1}$$

Part 4: Freq-Domain Transmission Lines

- **‘Looking in’**
- Resonance
- Half-Wave & Quarter-Wave Transformer
- Smith Charts
- VSWR
- Impedance Matching via QWT
- Impedance Matching via Short Stubbies

Impedance Looking In: Steady State

The remainder of the course deals with 'Input Impedance', i.e. $Z(d)$ at some interesting position d.

Input impedance: the impedance of the Thevenin resistor if the entire TL were replaced by this resistor.

Behavior of R_g and V_g is the same for both circuits!!!

Impedance Looking In: Steady State

In Thevenin-equivalent circuit, impedance Z_{in} has some voltage V_{in} over it and some current I_{in} running through it.

This corresponds to voltage and current at the input end of the TL!

Resonance

$$Z_{\text{in}} = Z_L \quad \text{when} \quad \beta\ell = n\pi \quad (n \geq 1)$$

$$\ell = \frac{n\lambda}{2}$$

What if $Z_L = 0$ and $Z_{in} = 0$? -> TL has length n lambda/2, n>=1

What if $Z_L = \infty$ and $Z_{in} = \infty$? -> TL has length n lambda/2, n>=1

Resonance

What if $Z_L = 0$ and $Z_{in} = \infty$? -> (2n+1) lambda/4, n>=0

What if $Z_L = \infty$ and $Z_{in} = 0$? -> (2n+1) lambda/4, n>= 0

Resonance

Type	Input	Output	T.L. Length
Series	Short	Short	$\frac{n\lambda}{2}$, n integer
Series	Short	Open	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Short	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Open	$\frac{n\lambda}{2}$, n integer

Problem: Resonance

Length of TL: 3m. $v = c$.
Load is shorted.

What input frequencies make Z_{in} equivalent to a short circuit?

$\ell = n\, \lambda/2$
$3 = n\, \lambda/2$
$\lambda = 6/n$
$f = v/\lambda = c\,/\,(6/n) = (c\, n)/6$
$f_n = 50\, n$ MHz

What input frequencies make Z_{in} equivalent to an open circuit?

$\ell = (2n + 1)\, \lambda/4$
$3 = (2n + 1)\, \lambda/4$
$\lambda = 12/(2n + 1)$
$f = v/\lambda = (c\,(2n + 1))/12$
$f_n = 25\,(2n + 1)$ MHz

Problem: Resonance

$v = c.f = 3 * 10^8$ Hz.
Load is open circuit.

What lengths of TL make Z_{in} equivalent to a short circuit?

$\cot(\beta\ell) = 0$
$\beta\ell = (2n + 1)\ \pi/2$
$\ell = (2n + 1)\ \lambda/4$
$n = 0, 1, 2, \ldots$

What lengths of TL make Z_{in} equivalent to an open circuit?

$\cot(\beta\ell) \to \infty$
$\beta\ell = n\pi$
$\ell = n\ \lambda/2$
$n = 1, 2, 3, \ldots$

Half-Wave Transformer

Allows you to 'crawl' up (or down) a TL in $\frac{\lambda}{2}$ steps.

$$\tilde{V}_{in} = -\tilde{V}_{out}$$
$$\tilde{I}_{in} = -\tilde{I}_{out}$$
$$Z_{in} = Z_{out}$$

Quarter-Wave Transformer

Allows you to 'crawl' up (or down) a TL in $\frac{\lambda}{4}$ steps.

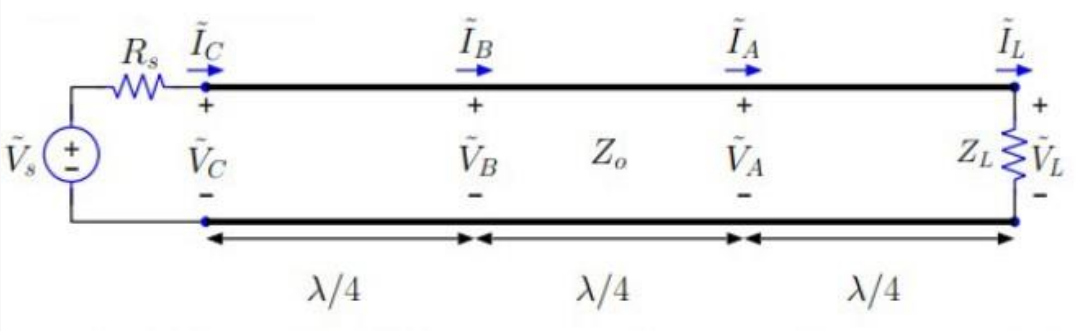
$$\tilde{V}_{in} = j\tilde{I}_{out}Z_0$$

$$\tilde{I}_{in} = \frac{j\tilde{V}_{out}}{Z_0}$$

$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

Problem: Transformers

$\tilde{V}_s = 5e^{j\frac{\pi}{6}}, R_s = 75\Omega$
$Z_0 = 25\Omega, Z_L = 50 + j50\Omega$

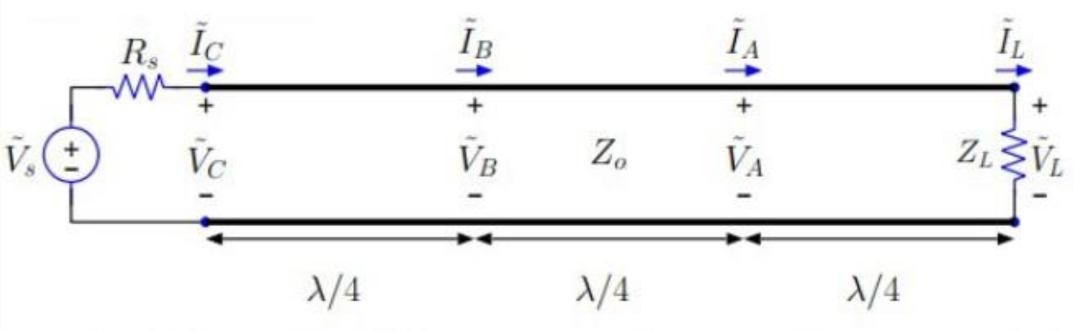


Find $Z_B = \frac{\tilde{V}_B}{\tilde{I}_B}$ and $Z_C = \frac{\tilde{V}_C}{\tilde{I}_C}$.
Find $\tilde{V}_C$, $\tilde{V}_A$, and $\tilde{I}_L$.
Find average power absorbed by Z_L.

B is half wavelength away from load, so use HWT, Z_B = Z_L
C is quarter wave away from B, so use QWT, so ZC = ZO^2 / ZO.
VC –> voltage divide between Rs and ZC
A is half wave away from input, = –VC
IL is quarter wave away from A, use QWT

Problem: Transformers

$\tilde{V}_s = 5e^{j\frac{\pi}{6}}$, $R_s = 75\Omega$
$Z_0 = 25\Omega$, $Z_L = 50 + j50\Omega$



Find $Z_B = \frac{\tilde{V}_B}{\tilde{I}_B}$ and $Z_C = \frac{\tilde{V}_C}{\tilde{I}_C}$.
Find $\tilde{V}_C$, $\tilde{V}_A$, and $\tilde{I}_L$.
Find average power absorbed by Z_L.

$$\tilde{V}_s = 5e^{j\pi/6}, \quad R_s = 75\ \Omega, \quad Z_0 = 25\ \Omega, \quad Z_L = 50 + j50\ \Omega$$

$$Z_B = Z_L = 50 + j50\ \Omega$$

$$Z_C = \frac{Z_0^2}{Z_B} = \frac{25^2}{50 + j50} = 6.25 - j6.25\ \Omega$$

$$\tilde{V}_C = \tilde{V}_s \frac{Z_C}{R_s + Z_C} \approx 0.542\angle(-10.6°)\ \text{V}$$

$$\tilde{V}_A = -\tilde{V}_C \approx 0.542\angle(169.4°)\ \text{V}$$

$$\tilde{I}_L = -j\frac{\tilde{V}_A}{Z_0} \approx 0.0217\angle(-100.6°)\ \text{A}$$

$$P_L = \left|\tilde{I}_L\right|^2 \Re\{Z_L\} \approx 2.35 \times 10^{-2}\ \text{W} = 23.5\ \text{mW}$$

Smith Charts!

Allows you to ‘crawl’ up (or down) any TL of any length!

Smith Charts: Intro

$$V(d) = V^+ e^{j\beta d} + \Gamma_L V^+ e^{-j\beta d}$$

$$I(d) = \frac{1}{Z_0}(V^+ e^{j\beta d} - \Gamma_L V^+ e^{-j\beta d})$$

$$V(d) = V^+ e^{j\beta d}(1 + \Gamma_L e^{-2j\beta d})$$

$$I(d) = \frac{V^+ e^{j\beta d}}{Z_0}(1 - \Gamma_L e^{-2j\beta d})$$

Smith Charts: Intro

$$\Gamma(d) = \Gamma_L e^{-2j\beta d}$$
$$V(d) = V^+ e^{j\beta d}\big(1 + \Gamma(d)\big)$$
$$I(d) = \frac{V^+ e^{j\beta d}}{Z_0}\big(1 - \Gamma(d)\big)$$

$$Z(d) = \frac{V(d)}{I(d)} = \frac{Z_0\big(1 + \Gamma(d)\big)}{1 - \Gamma(d)}$$

$$z(d) = \frac{\big(1 + \Gamma(d)\big)}{1 - \Gamma(d)}$$

Pivotal Point 2

What does the leftmost point of the Smith Chart correspond to in terms of Γ?
Gamma = -1

What does the leftmost point of the Smith Chart correspond to in terms of z?
z=0
What does the leftmost point of the Smith Chart correspond to as a load?
Load is a short

Pivotal Point 3

What does the rightmost point of the Smith Chart correspond to in terms of Γ?
Gamma=1
What does the rightmost point of the Smith Chart correspond to in terms of z?
z=infty
What does the rightmost point of the Smith Chart correspond to as a load?
Load is open

Devious Details 1

April the Fool tells you that at the load end of the transmission line she's at angle 0 on the Smith Chart. She then walks 3 meters toward the input and asks what angle we're at now.

What information are you missing to solve this problem? -> WAVELENGTH

Why is it important? -> electrical length

Devious Details 2

April the Fool starts at the load and walks $\lambda/2$ distance towards the input. How many degrees did she walk through on the Smith Chart? Why? 360!

$$\Gamma\left(\frac{\lambda}{2}\right) = \Gamma_L$$

Devious Details 3

Sasha receives a transmission line problem with some load impedance and some intrinsic impedance. She first normalizes the load impedance, enters it on the Smith Chart, and then draws a constant Γ circle that passes through the origin and the point she drew.

Why did she do each of these three steps?

Devious Details 4

Eric finds the normalized load impedance of some transmission line setup. He enters it on the Smith Chart and finds the phase and magnitude of Γ at that point. What is the significance of this Γ?

Devious Details 5

When travelling towards the source Sasha always rotates clockwise along the Smith Chart. Why?
If Sasha instead travels towards the load, then which direction should she rotate?

Gamma (d) = Gamma_L e^(-j2beta d)

Devious Details 6

Does the Smith Chart yield **steady state** solutions, transient solutions, or both?

Example Smith Chart

TL length = 5m.
$v = c, f = 100\text{MHz}.$
$Z_L = 150 + \text{j}150\Omega, Z_0 = 50\Omega, R_g = 25\Omega.$
$\tilde{V}_g = 10e^{\frac{j\pi}{4}}$ V
Find Γ_L, Z_{in}, $\tilde{V}_{in}$, $\tilde{I}_{in}$. How would you find $\tilde{V}(0)$?

Gamma L is done
z_in is about 0.26-0.8j. Then Z_in = z_in * Z0. Z_in done
Vin, Iin free

Multiline TL: Parallel Resistor

Multiline TL: Parallel Resistor

Multiline TL: Parallel Resistor

Multiline TL: Parallel Resistor

VSWR

$$\tilde{V}(d) = \tilde{V}^{+} e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

What is the maximum magnitude of $\tilde{V}(d)$?
What is the minimum magnitude of $\tilde{V}(d)$?

VSWR

$$\tilde{V}(d) = \tilde{V}^{+} e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

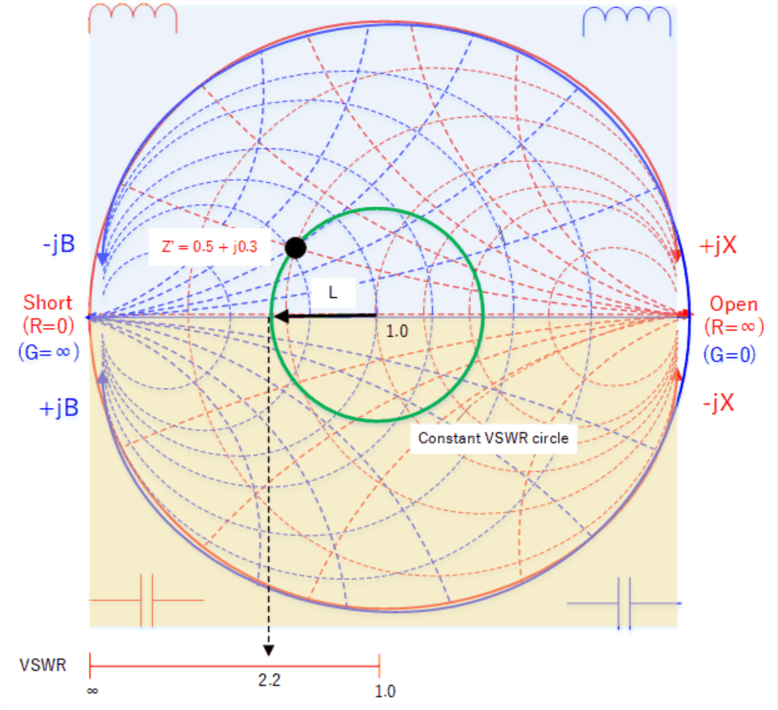
$$\text{VSWR} = \frac{\left|\tilde{V}(d)\right|_{\max}}{\left|\tilde{V}(d)\right|_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

VSWR

Where is VSWR on a Smith Chart?

On the Smith Chart, VSWR is the **circle around the center with radius |Γ|**.

|Γ|=0 → **perfect match** (no reflection)
|Γ|=1 → **total reflection** (short or open)



Larger radius → bigger mismatch → higher VSWR
Small radius → close to center → VSWR close to 1 (good match)

VSWR Demo

https://www.desmos.com/calculator/hg96trh9c9

VSWR

What are the extreme values that VSWR can take on?
1 -> matched impedance (Gamma_L = 0)
Infinity -> |Gamma_L| = 1 = perfect standing wave

What situations do they represent in real life?

Impedance Matching: Motivation

$$\tilde{V}(d) = \tilde{V}^+ e^{j\beta d} + \tilde{V}^- e^{-j\beta d}$$

$$\tilde{V}(d) = \tilde{V}^+ e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

If $\Gamma_l = 0$, the parentheses become 1. meaning **no reflections, no standing waves, maximum power transfer**

Impedance Matching: Quarter-Wave Transformer

Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 125 + j75\Omega$

A quarter-wave transformer will be inserted on the TL at some distance d away from the load. Find d and the optimal intrinsic impedance of the line to be inserted Z_q such that the load is a perfect match.

Impedance Matching: Quarter-Wave Transformer

This method arises from how the Smith Chart works!!

While you can write down the exact steps you should follow to solve this and regurgitate on the exam, it is highly recommended that you instead understand why you are doing what you are doing.

Each step has a purpose. Understanding this purpose will help you deal with the countless variations that they could throw at you.

- See HW14 Q4

Impedance Matching: Quarter-Wave Transformer

1. Normalize the load impedance and plot it on the Smith Chart. The Smith Chart deals with normalized impedances.
2. Walk to the source towards the horizontal axis of the Smith Chart. This gives us a real z_{out}, which is useful for the next step. The distance traveled is d.

3. Denormalize z_{out} to get Z_{out}. The quarter-wave transform formulae deal with denormalized impedances. Both Z_{in} and Z_{out} are now known!
4. Use the quarter-wave transform formula to find Z_q: $Z_q^2 = Z_{in}Z_{out}$.

Impedance Matching: Open-Stub

Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 125 + j75\Omega$

An open stub with characteristic impedance $Z_1 = 50\Omega$ will be inserted in parallel on the TL at some distance d away from the load. Find shortest d and the length of an open-circuited stub d_s required such that the load is a perfect match.

Travelled from 0.464 to 0.164, so distance **d is 0.2 lambda**.
y1 is 1+1.35j. Y1 = 1/50 (1+1.35j)
Need to cancel out Y = 1/50(1.35j), or stub y_in needs to be -1.35j
ds = 0.352 lambda.

Impedance Matching: Open-Stub

1. Normalize the load impedance and plot it on the Smith Chart. The Smith Chart deals with normalized impedances.
2. Convert to normalized load admittance and plot it on the Smith Chart. We're adding a stub in parallel. Admittances add when combined in parallel. This step is unnecessary but makes the math easier.
3. Find the first intersection between the constant gamma circle and the r = 1 circle.
4. Calculate distance from load admittance to intersection point found in step 3. This is the distance to insertion point d.
5. Denormalize to yield denormalized admittance. The complex part needs to be cancelled out.
6. On another Smith Chart, start at the load end of the stub. Rotate until the denormalized admittance cancels the denormalized admittance found in step 5 when adding the two denormalized admittances together.
7. Calculate distance from the stub's load to the point in step 6. This is the length of the stub d_s.

Final Exam Notes

You are allowed one 8.5″x11″ (letter size) cheatsheet, both sides.
It must be handwritten.

Bring:

- Your cheatsheet. You will be submitting this.
- A pencil (pen not recommended)
- A compass (to draw circles)
- A protractor
- A straightedge

Exam 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\hat{n}\cdot\left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho\, dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}\, dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Exam 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$\hat{n} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$\hat{n} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s$$

$$\hat{n} \times \left(\vec{M}_1 - \vec{M}_2\right) = \vec{J}_{b,s}$$

Exam 3 equations, in one place

Waves:

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}\hat{z}$$

TLs:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E} + \frac{1}{2}\mu\vec{H}\cdot\vec{H}\right) + \nabla\cdot\vec{S} + \vec{J}\cdot\vec{E} = 0$$

Final exam equations, in one place

Multiline, from TL j to TL k

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$
$$\tau_{jk} = 1 + \Gamma_{jk}$$

Smith Charts

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$
$$z(d) = \frac{1+\Gamma(\mathrm{d})}{1-\Gamma(\mathrm{d})}$$
$$\text{VSWR} = \frac{1+|\Gamma(\mathrm{d})|}{1-|\Gamma(\mathrm{d})|}$$
$$z(d) = \frac{Z(d)}{Z_0}$$
$$y(d) = \frac{1}{z(d)}$$

Quarter-Wave Transformer

$$\tilde{V}_{in} = j\tilde{I}_{out} Z_0$$
$$\tilde{I}_{in} = \frac{j\tilde{V}_{out}}{Z_0}$$
$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

Half-Wave Transformer

$$\tilde{V}_{in} = -\tilde{V}_{out}$$
$$\tilde{I}_{in} = -\tilde{I}_{out}$$
$$Z_{in} = Z_{out}$$

$$\tilde{I}(d) = \frac{1}{Z_0}(\tilde{V}^+ e^{j\beta d} - \tilde{V}^- e^{-j\beta d})$$
$$\tilde{I}(d) = \frac{1}{Z_0}(\tilde{V}^+ e^{j\beta d}(1 - \Gamma_L e^{-j2\beta d}))$$

Resonance:

Type	Input	Output	T.L. Length
Series	Short	Short	$\frac{n\lambda}{2}$, n integer
Series	Short	Open	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Short	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Open	$\frac{n\lambda}{2}$, n integer

$$\tilde{V}(d) = \tilde{V}^+ e^{j\beta d} + \tilde{V}^- e^{-j\beta d}$$
$$\tilde{V}(d) = \tilde{V}^+ e^{j\beta d}(1 + \Gamma_L e^{-j2\beta d})$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_S: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Characteristic impedance Z: Ohm

Thank you

for a great semester ☺