ECE329: Exam 1 Review Session

9/25/2025

Exam 1 Content

- Vector Calculus
- Coulomb's Law and Lorentz Force
- Gauss's Law & electric flux
- Electrostatic potential
- Boundary Conditions
- Conductors
- Dielectrics
- Capacitance & Conductance
- Charge flux

1.

Math ☹

Coordinate Systems

Cartesian 3D

Cylindrical

Spherical

Line, Surface, & Volume integrals

Volume

Line

Surface

Divergence & Divergence Theorem

Divergence = How much the field is DIVERGING at a certain point.

- Notation: $\nabla \cdot \vec{D}$
- Input: Vector field
- Output: Scalar field

Divergence Theorem: $\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$

Curl & Stoke's Theorem

Curl = How much the field is CURLING around a certain point.

- Notation: $\nabla \times \vec{E}$
- Input: Vector field
- Output: Vector field, with direction indicating how the field right-hand curls around.

Stoke's Theorem: $\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$

5. (12 pts) For $\mathbf{E} = yz\hat{x} + (y + zx)\hat{y} + xy\hat{z}$ V/m

a) Evaluate the potential difference $V_A - V_B$ for $\mathbf{A} = (2, 1, 1)$ and $\mathbf{B} = (1, 4, \frac{1}{2})$

Gradient & Gradient Theorem

Gradient = How much the scalar function changes (or its GRADE).

- Notation: ∇V
- Input: Scalar field
- Output: Vector field, with direction indicating steepest **uphill**.

Fundamental theorem of calculus: $\int_a^b f'(t) \cdot dt = f(b) - f(a)$

Similarly:

Gradient theorem: $\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$ = Voltage gain from a to b.

Laplacian

Laplacian (scalar) = How much the scalar function LAPLACES.

Laplacian (scalar) = How much the rate of change of the scalar function varies (aka stress).

- Notation: $\nabla^2 V = \nabla \cdot \nabla V$
- Input: Scalar field
- Output: Scalar field

P.S.: Helmholtz Theorem

A vector field $\vec{E}$ is specified completely by its divergence and curl.

2.

Physics ☺

All* the concepts

Q $\rho_f = \rho$ $\vec{D}$ ρ_s

C $\rho_{b,s}$

V $\vec{E}$ $\vec{P}$ ρ_b

In the beginning…

There exists positive free electric charge and negative free electric charge.

Coulomb's Law:

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon r^2}\hat{r}$$

Fields

Electric field [Newtons/Coulomb] OR [Volts/meter]: the effect that a charge has on its surroundings.

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon r^2}\hat{r}$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E}$$

Superposition

$$\vec{F} = q_1 \sum^{n} \frac{q_n}{4\pi\epsilon|\vec{r}_n|^2} \frac{\vec{r}_n}{|\vec{r}_n|} \qquad \vec{E} = \sum^{n} \frac{q_n}{4\pi\epsilon|\vec{r}_n|^2} \frac{\vec{r}_n}{|\vec{r}_n|}$$

Summation can become an integral!

Electric Field Flux

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2}\hat{r}$$

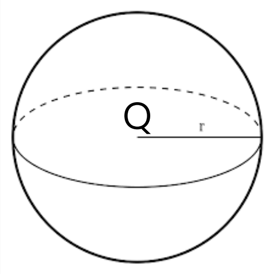
'Electric field flux', or '**Flux**' [N*m^2/C] OR [V*m].

$$\iint \vec{E} \cdot d\vec{S}$$

Gauss's Law: Electric

For closed surfaces only:

- If there is more electric field flux,
- Then there is more electric field flowing out of the surface,
- Then, more free charge must be enclosed in the surface.



This is **Gauss's Law**!

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

2. Two parts of the following question are independent:

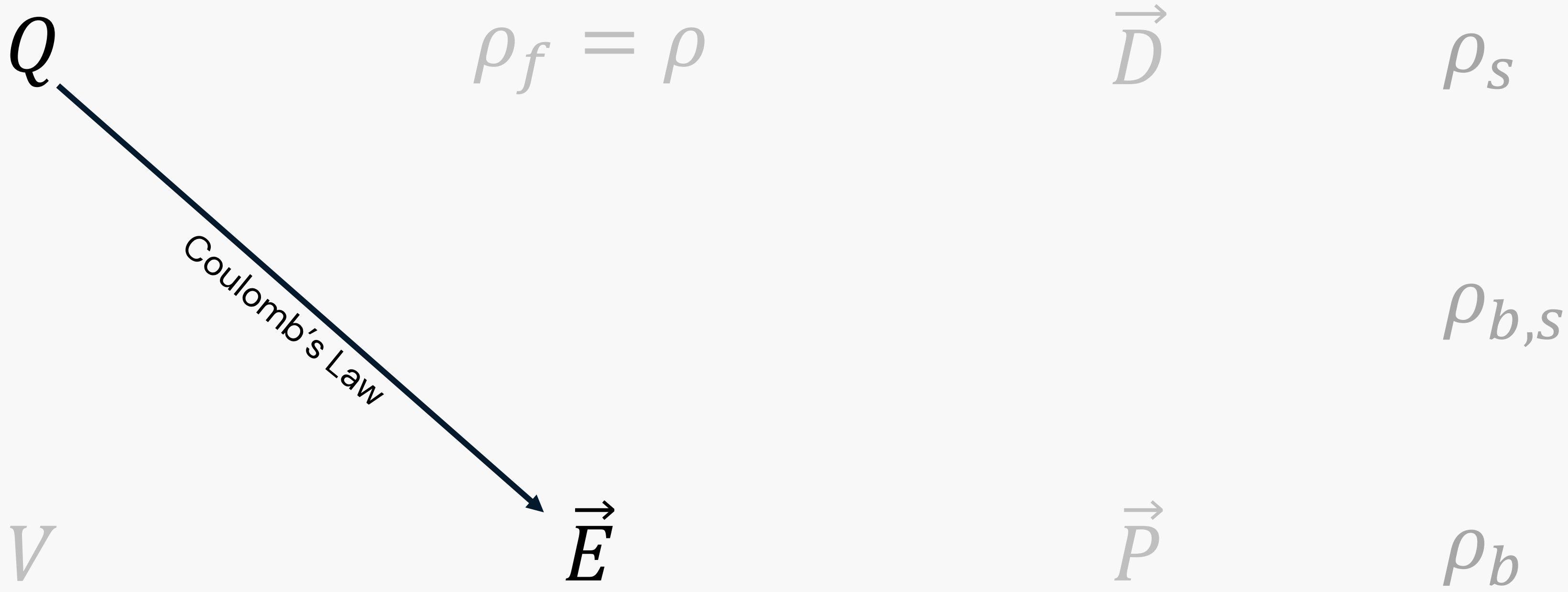
a) Consider an infinite slab of width $W = 2$ m occupying the region $|x| < 1$ m. The charge density within the slab is $\rho = 4$ C/m^3 and it is zero outside the slab.

i. (4 pts) Determine the electric field vector $\mathbf{E}(x, 0, 0)$ for $x = 2$ m and $x = -2$ m.

ii. (4 pts) Determine the electric field vector $\mathbf{E}(x, 0, 0)$ for $-1 < x < 1$ m within the slab.

summer20

All* the concepts



Gauss's Law: Magnetic

There do not exist positive magnetic charges or negative magnetic charges!

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

Displacement Field

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

Recall vector field $\vec{D} = \epsilon \vec{E}$.

Electric displacement field: $\vec{D}$ [C/m^2]
Electric displacement flux: Closed surface integral of vector field $\vec{D}$.

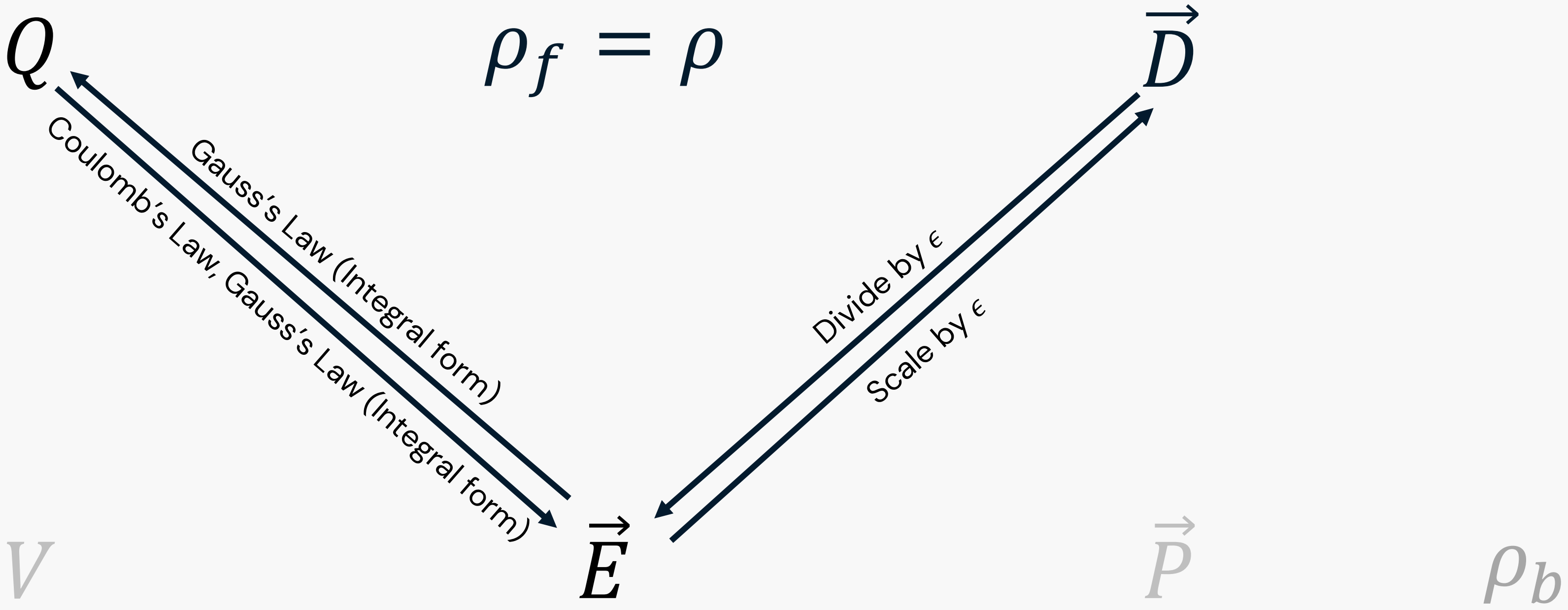
$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

Charge Density

Charge density: ρ [C/m^3]

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = \oiint \vec{D} \cdot d\vec{S} = \iiint \rho dV = Q_{\text{enclosed}}$$

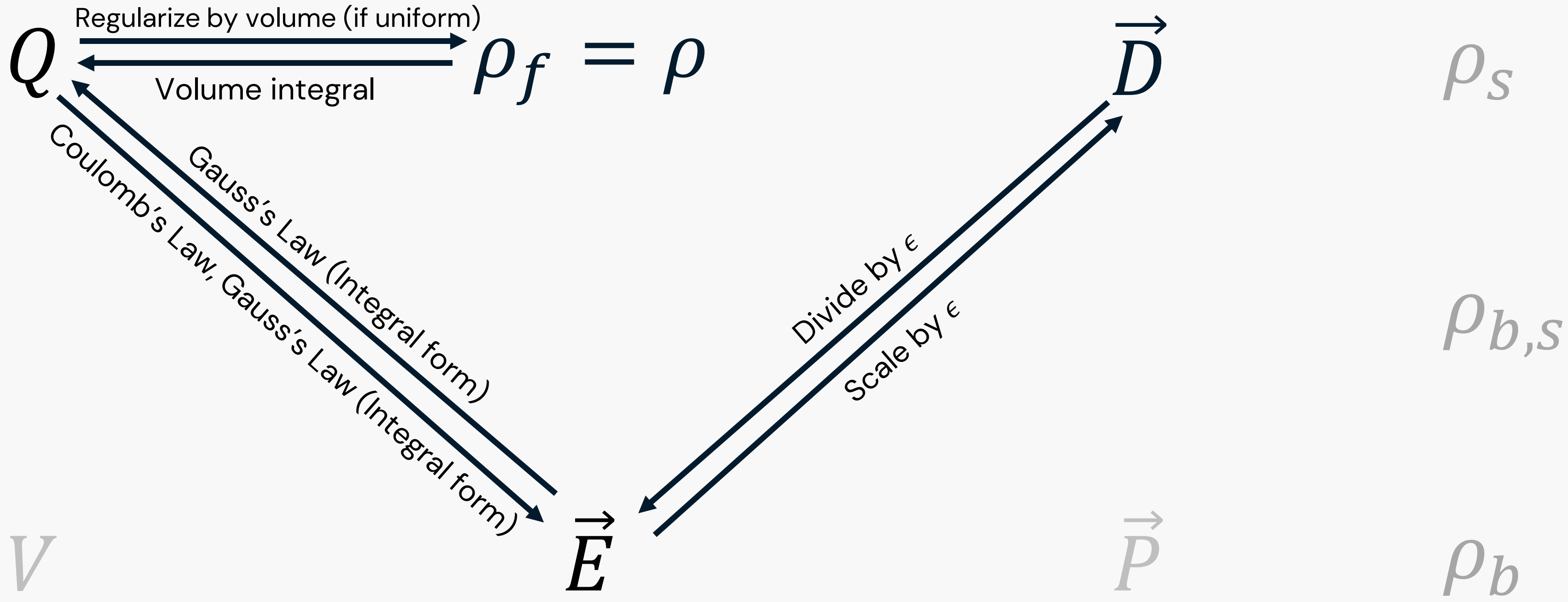
All* the concepts



Gauss's Law: Differential form

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \rho_{\text{enclosed}} dV \qquad \oiint \vec{B} \cdot d\vec{S} = 0$$

All* the concepts



P fields: The intuition

P fields: The math

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: The defining equation.

Assumption: Dielectric is ‘isotropic’, so $\vec{P}$ is collinear to $\vec{E}$. Then:

- $\vec{P} = \epsilon_0 \chi_e \vec{E}$ with **electric susceptibility** $\chi_e \geq 0$ nearly always in this class.
- Let **electric permittivity** be $\epsilon = \epsilon_0(1 + \chi_e)$.
- Let **relative electric permittivity** be $\epsilon_r = 1 + \chi_e$
- $\vec{D} = \epsilon \vec{E}$

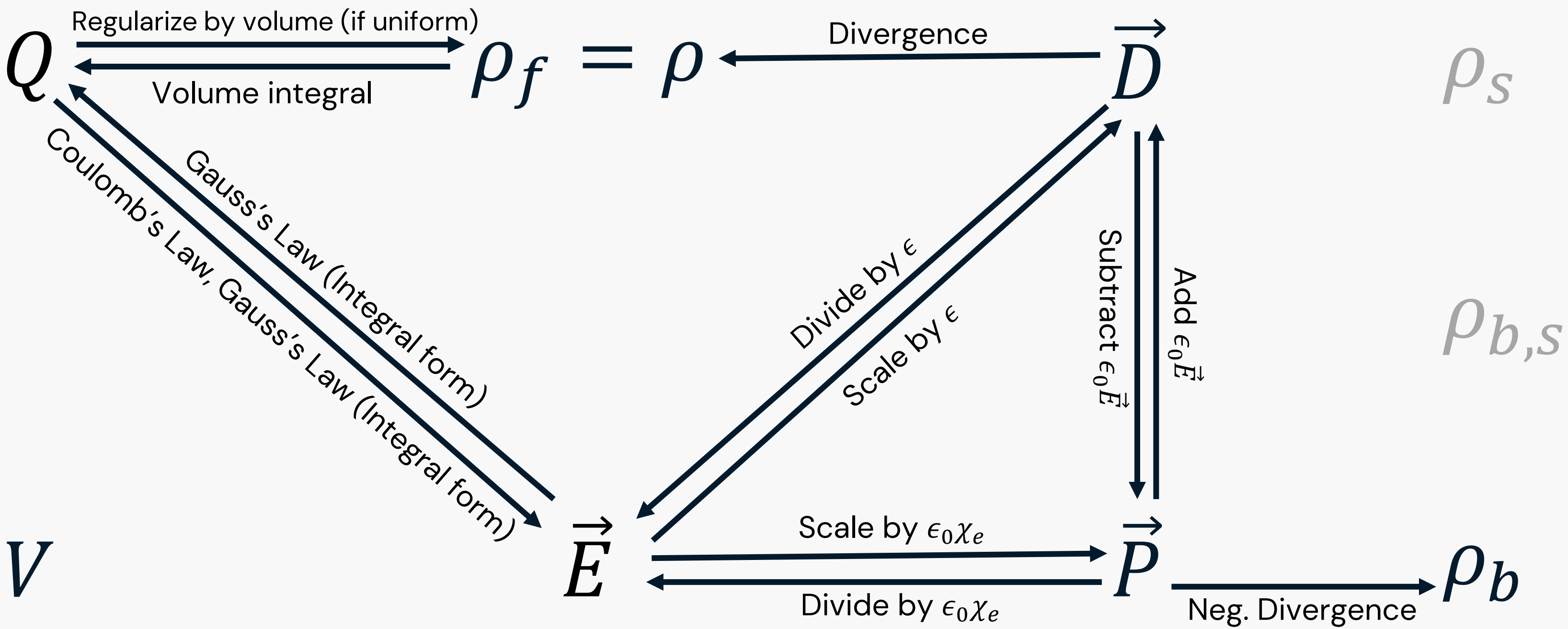
P fields: The math

Divergences:

- Gauss's Law: $\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b$
- Gauss's Law: $\nabla \cdot \vec{D} = \rho_f = \rho$

- Therefore, $\rho_b = -\nabla \cdot \vec{P}$

All* the concepts



Conservative Fields

The following are equivalent for vector field $\vec{E}$:

- $\nabla \times \vec{E} = 0$
- $\vec{E}$ is conservative
- $\oint \vec{E} \cdot d\vec{l} = 0$
- $\int_a^b \vec{E} \cdot d\vec{l}$ is path-independent
- $\vec{E} = \nabla V$ for some scalar field V
- $\vec{E}$ field arises from electrostatics

Summer 18

a) Consider the electric field $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1 = \hat{x}\sin(\pi x/2)$ V/m and $\mathbf{E}_2 = \hat{x}\sin(\pi y/2)$ V/m.

i. (4 pts) Which one of $\mathbf{E}_1$ and $\mathbf{E}_2$ satisfies the electrostatic field conditions? Explain.

Electrostatic Potential

Work done by the electric field to move a charge from a to b, causing drop in electrostatic potential energy U:

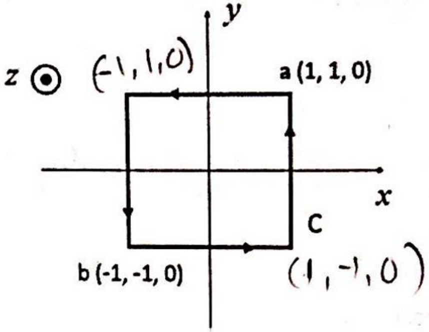
$$-\Delta U = -[U(b) - U(a)] = \int_a^b q\vec{E} \cdot d\vec{l}$$

Volts = work per unit charge (take $q = 1$).
U/q = electrostatic potential energy per unit charge = V = **electrostatic potential**:

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

Example

2. (25 points) Considering elelctric field **E** in free space and path C indicated in figure below, answer the following questions:



Spring 18

a) If the electric field is $\mathbf{E} = 2\hat{x} - 3\hat{y} + 5\hat{z}$ V/m,

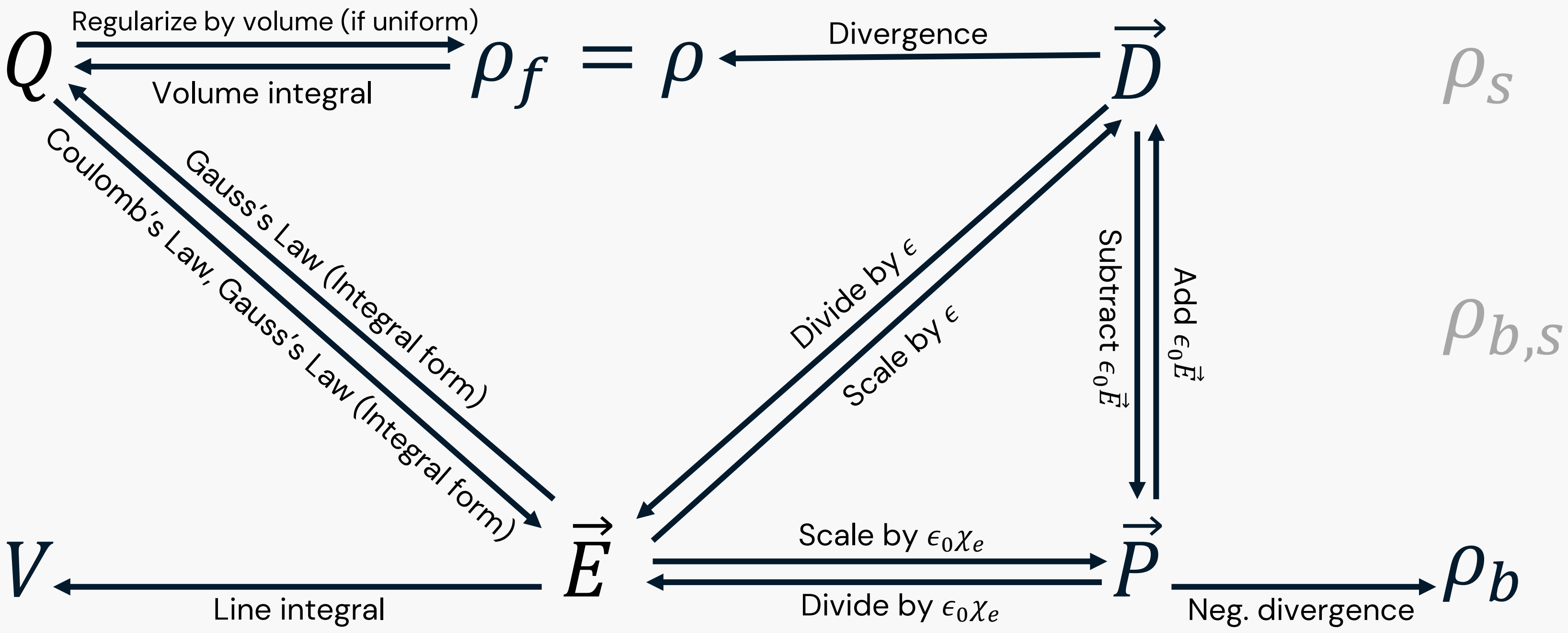
i. (7 pts) what is the circulation $\oint \mathbf{E} \cdot d\mathbf{l}$ around path C? Show your work.

ii. (7 pts) what is the electrostatic potential drop from point a to b (i.e. $V(a) - V(b)$)?

E to V

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

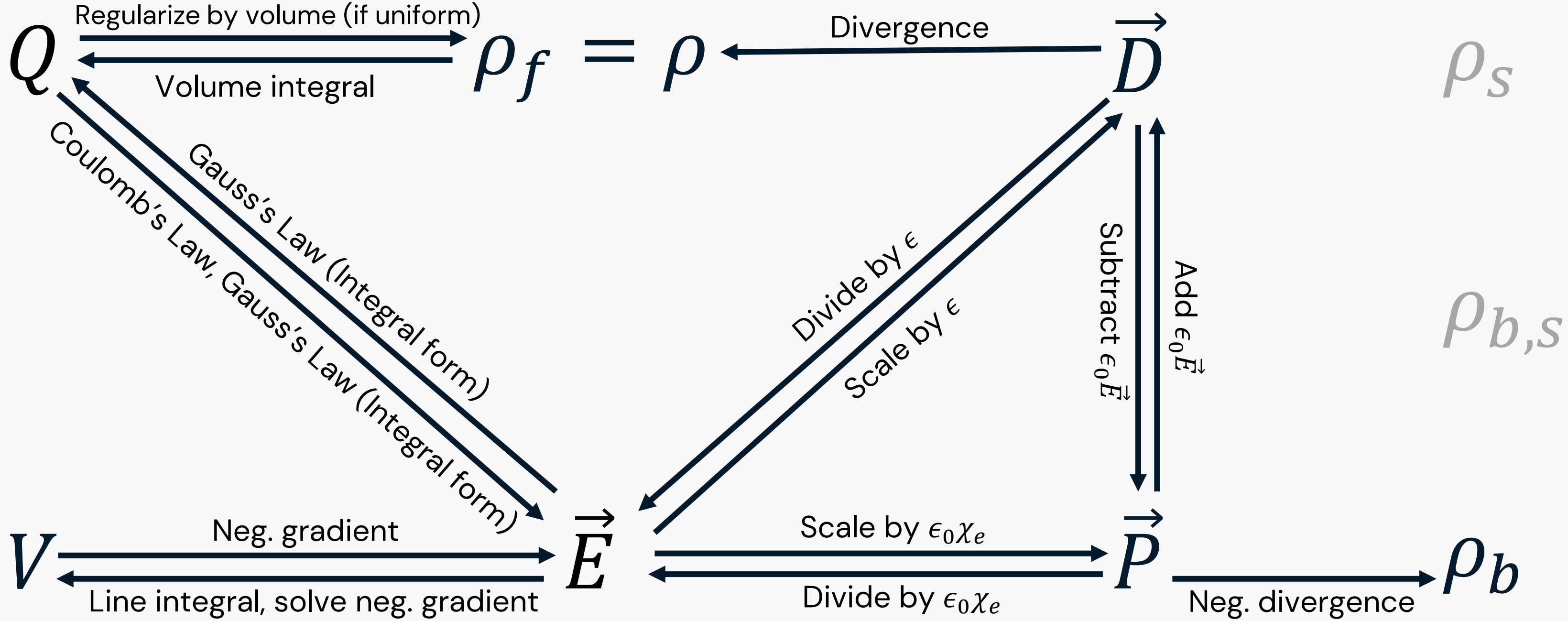
All* the concepts



Poisson’s Equation

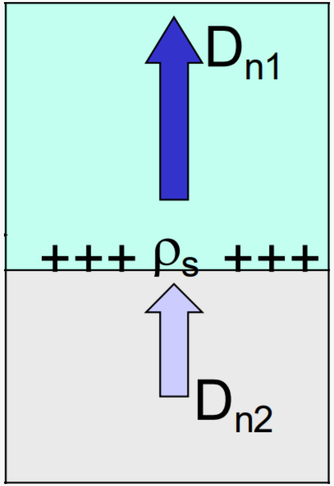
Laplace’s Equation

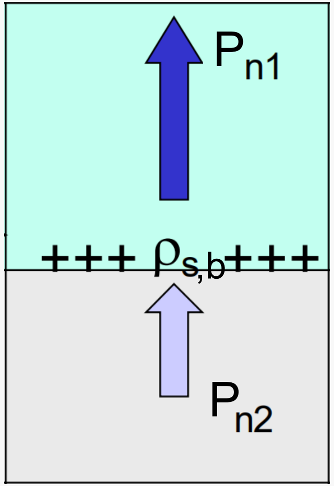
All* the concepts

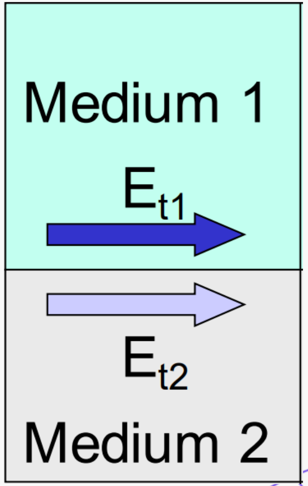


Boundary Conditions

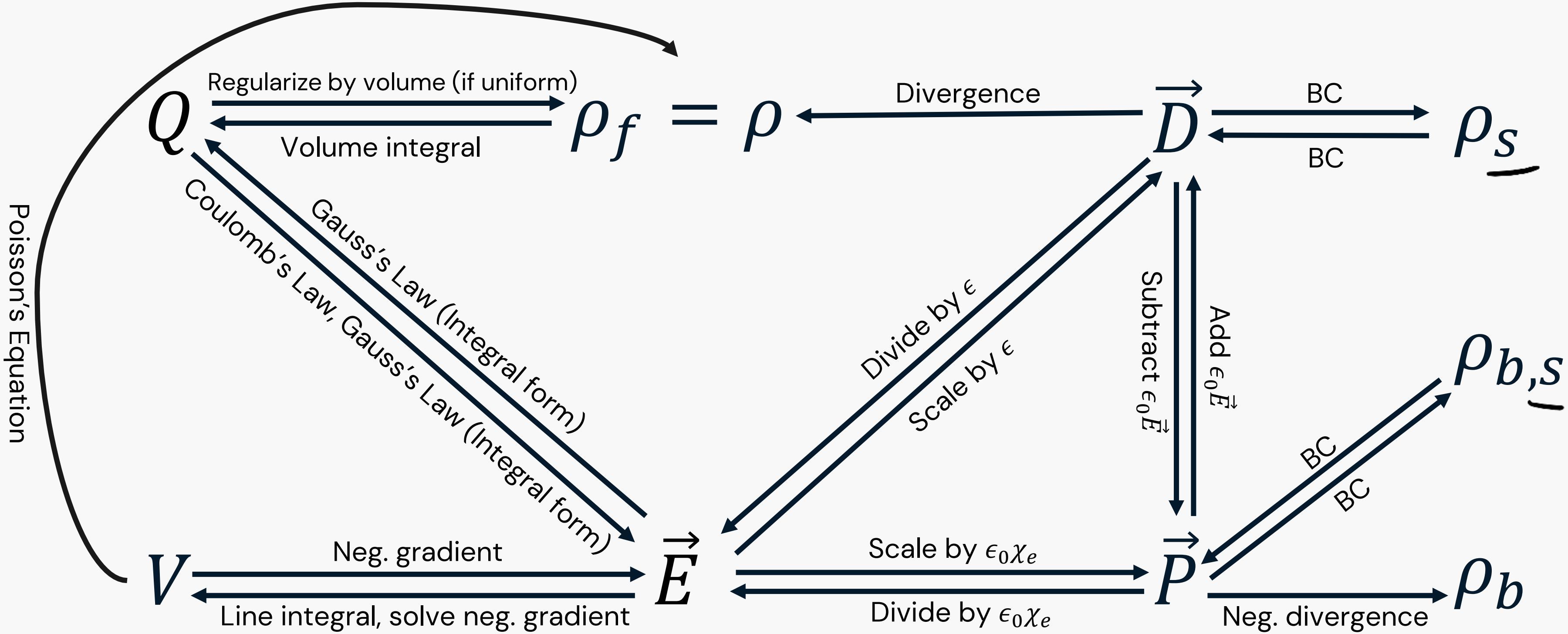
$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s \quad \hat{n} \cdot \left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s} \quad \hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$







All* the concepts



HW3

6. Two dielectric media with permittivities $\epsilon_1 = \epsilon_0$ and $\epsilon_2 = 4\epsilon_0$ are separated by a charge free boundary. The electric field in medium 1 has magnitude of 10 $\frac{\text{V}}{\text{m}}$ at an angle 30 degrees from the normal.

a) Find the magnitude and direction of the electric field in medium 2.

b) Suppose the electric field in medium 1 is the same, but the electric field in medium 2 is now $10\frac{\text{V}}{\text{m}}$. Find the surface charge on the boundary.

c) Suppose the boundary is charge-free and medium 2 is now a slab of finite thickness with medium 3 ($\epsilon_3 = \epsilon_0$) on the other side. What is the angle from normal of the electric field in medium 3?

Capacitance

Capacitance: the ability of something to collect and store energy in the form of electrical charge.

This energy is stored as opposite electric charges being held apart (and thus creating a difference in electric potential, aka a voltage drop).

$$Q = CV \qquad G = \frac{\sigma}{\epsilon} C \qquad R = \frac{1}{G}$$

3. Two parts of the following question are independent:

a) (15 pts) Consider the following *spherically* symmetric configuration of composite materials in steady-state equilibrium:

i. The region defined by $r \leq 1$ m, where $r = \sqrt{x^2 + y^2 + z^2}$ is the radial distance from the center of the spherical configuration, has $\epsilon = \epsilon_o$, $\sigma = 10^6$ S/m, and holds a net charge of $Q = +4$ C distributed uniformly over its spherical surface.

ii. A perfect dielectric shell with $\epsilon = 2\epsilon_o$ occupies the region $1 < r < 2$ m.

iii. Region $2 \leq r \leq 3$ m has the same material properties as region $r \leq 1$ m and holds zero net charge.

iv. Region $r > 3$ m is occupied by free space. In all four regions we have $\mu = \mu_o$.

Determine, in all four regions, (a) **D**, (b) **E** and (c) **P**, *and* (d) the surface charge densities, in C/m^2 units, at each of the three material boundaries at $r = 1$, 2, and 3 m. **Hint:** Make use of Gauss's law in integral form, $\oint_S \mathbf{D} \cdot dS = \int_V \rho dV$, with $\mathbf{D} = \epsilon\mathbf{E} = \epsilon_o\mathbf{E} + \mathbf{P}$, and a crucial fact about steady-state fields within conducting materials.

Summer 18

3.

Some random stuff

P.S.: Maxwell's Equations

Maxwell collected these equations.
Don't worry about where it came from.

$\nabla \cdot \vec{D} = \rho$	Gauss's Law (electric)
$\nabla \cdot \vec{B} = 0$	Gauss's Law (magnetic)
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Ampere's Law (with Maxwell's correction)

P.S.: Lorentz Force

Currents create magnetic fields for some reason.
Magnetic fields also exert force on moving charges for some reason.

So, here’s an equation to describe that.

Lorentz Force equation

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

P.S. Current Density

Current: movement of charge, denoted I [Amps] = [Coulomb/second].

Charge flux: movement of charge through a surface.

Current density: Denoted $\vec{J}$ [Amps/meter^2]. Integrating current density over a surface yields charge flux!

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

P.S. Current Flux

$$\oiint \overrightarrow{D} \cdot d\vec{S} = \iiint \rho dV$$

$$\oiint \vec{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint \rho dV$$

P.S. Conductors: The intuition

P.S. Conductors: The math

Described by σ, aka conductivity (units: Siemens/meter)

What to know:

- $\vec{J} = \sigma\vec{E}$ (Ohm's Law)

Assumption: We are dealing with electrostatics, i.e. steady-state.

- If $\sigma \neq 0$, material has zero internal fields and finite surface charge densities.

Exam 1 Prep

- Make your own 4”x6” notecard. Do not blindly copy other people’s notecards. Make sure you understand what you are writing on your notecard.

- Review HWs 1–4.
- Do the tutorial problems and review the solutions. These also contain rubrics, which will be similar to how your exam will be graded.
- Do the practice exams on course website.
- Read over Professor Kudeki’s notes if you have time. They are a bit dense.

Exam 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n}\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n}\cdot(\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho\, dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}\, dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Units

Charge Q: C
Electric field $\vec{E}$: N/C or V/m
Displacement field $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic field $\vec{B}$: T or Wb/m^2
Charge density ρ: C/m^3
Surface charge density ρ_S: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Good Luck!

Any questions?