ECE329: Exam 3 Review Session

April 16th, 2025

Exam 3 Content

- Poynting Theorem
- Phasors
- TEM Wave Propagation in Material Media
- Polarization
- Standing Waves
- Bounce Diagrams & TLs

Poynting Vector & Theorem

$\vec{S} = \vec{E} \times \vec{H}$. Units: W/m^2. “Instantaneous” power per unit area passing through surface in direction of $\vec{S}$

Energy unit volume balance equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E}+\frac{1}{2}\mu\vec{H}\cdot\vec{H}\right)+\nabla\cdot\vec{S}+\vec{J}\cdot\vec{E}=0$$

Phasor Notation

$$\vec{E}(x,t) = A\cos(\omega t - \beta x)\hat{z} = \mathrm{Re}\left\{Ae^{j\omega t}e^{-j\beta x}\hat{z}\right\} \longleftrightarrow Ae^{-j\beta x}\hat{z} = \tilde{E}(x)$$

Time domain

Phasor

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

Time-Averaging

If TEM wave $\vec{E}$ is cosinusoidal,

- Then corresponding $\vec{H}$ is cosinusoidal
- Then phasors $\tilde{E}$ and $\widetilde{H}$ exist
- Also, $\vec{S} = \vec{E} \times \vec{H}$ (instantaneous power per unit area) is cosinusoidal squared
- We can write $\vec{S}$ in phasor form too: $\tilde{S} = \tilde{E} \times \widetilde{H}^*$
- Time-average of cosinusoidal squared is ½ of the magnitude of the cosinusoidal:

$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}\{\tilde{E} \times \widetilde{H}^*\}$$

Wave Equation in Material Media

Assumptions: $\rho = 0,\ \vec{J} = 0$, i.e. region is a source-free. $\sigma \neq 0$ now!

Wave equation: $\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$. Solutions are TEM waves.
$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

General form of cosinusoidal solution:

Getting H from E

$$\vec{E} = Ae^{\alpha y}\cos(\omega t + \beta y + \phi)\,\hat{x} \text{ [V/m]}$$

$$\vec{E} = Ae^{\alpha y}e^{j\beta y}e^{j\phi}\hat{x} \text{ [V/m]}$$

Getting E from H

$$\vec{H} = Ae^{\alpha y} \cos(\omega t + \beta y + \phi)\, \hat{x} \text{ [A/m]}$$

$$\vec{H} = Ae^{\alpha y} e^{j\beta y} e^{j\phi} \hat{x} \text{ [A/m]}$$

EXACT Formulae

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

APPROXIMATE Formulae

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

Wave Polarization

Polarization is how the tip of $\vec{E}$ varies over time.

Linear: In phase
Circular: 90 degrees out-of-phase with equal magnitude
- Right-Handed
- Left-Handed

Elliptical: Anything else

Wave Polarization

Example of linear polarization: $\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\hat{x}$

Example of right-handed circular polarization (RHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Example of left-handed circular polarization (LHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} - 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Wave Polarization

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + \cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{3}\right)\hat{y}$$

What about this?

$$\vec{\mathrm{E}} = 2\cos(\omega t - \beta z)\,\hat{x} + 4\cos(\omega t - \beta z)\hat{y}$$

Wave Polarization Example

Find the polarization of:

$$\vec{\mathrm{E}} = 3\cos(\omega t + \beta x)\,\hat{z} - 3\cos\left(\omega t + \beta x - \frac{\pi}{2}\right)\hat{y}$$

Wave Polarization Example

Find the polarization of:

$$\vec{\mathrm{E}} = 3\cos(\omega t + \beta x)\,\hat{z} - 3\cos\left(\omega t + \beta x + \frac{\pi}{2}\right)\hat{y}$$

Wave Reflection & Transmission

TEM wave incident normally on a boundary

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Reflection & Transmission Coefficients

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_t(x) =$$

$$\tilde{E}_r(x) =$$

$$\tilde{H}_t(x) =$$

$$\tilde{H}_r(x) =$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Summary

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_r(x) = E_0 \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{y}$$

$$\tilde{H}_r(x) = -\frac{E_0}{\eta_1} \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{z}$$

$$\tilde{E}_t(x) = E_0 \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{y}$$

$$\tilde{H}_t(x) = \frac{E_0}{\eta_2} \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Coefficients

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Check our work:

1. What if $\eta_1 = \eta_2$?

2. What if $\eta_2 = 0$?

Standing Waves (dielectric to PEC)

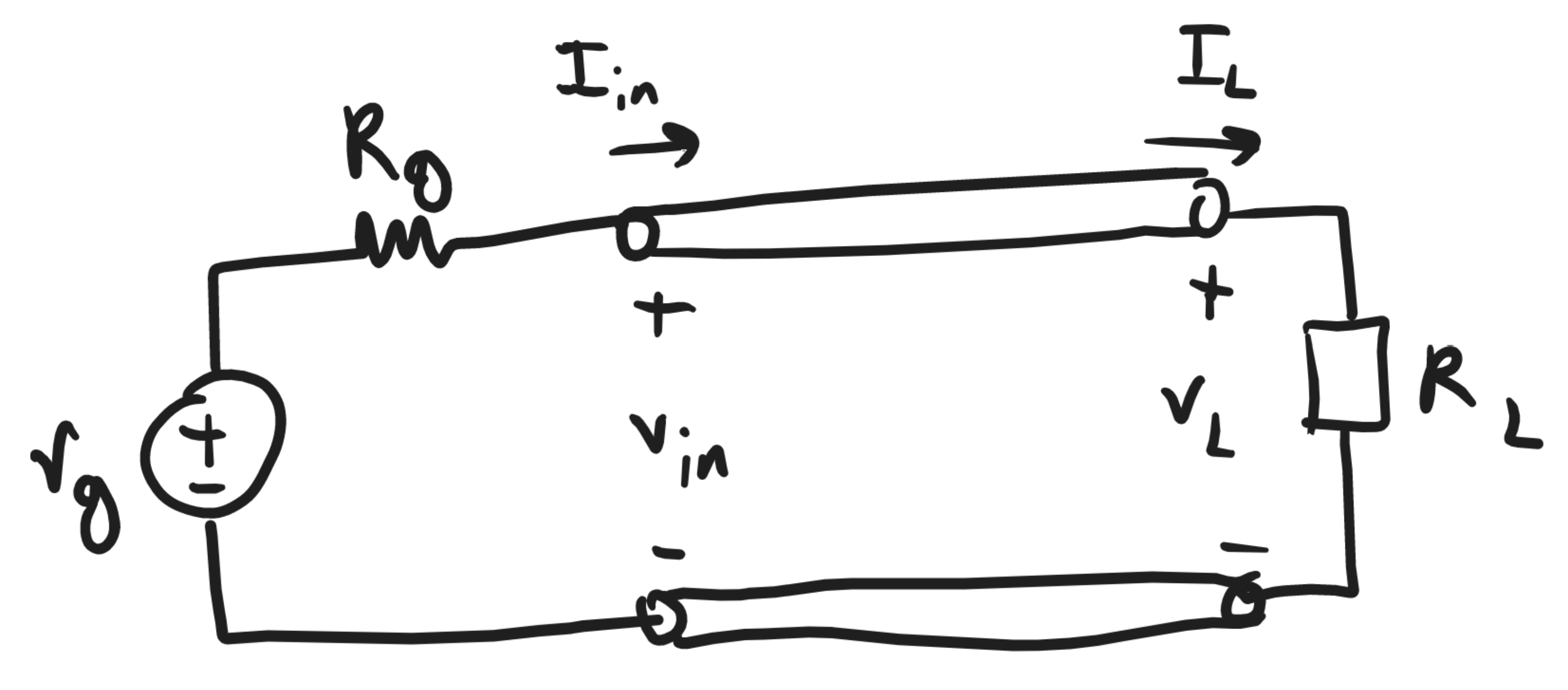
$$\tilde{E}_i(y) = -E_0 e^{-j\beta_1 y}\hat{x}$$

$$\tilde{E}_r(y) = E_0 e^{j\beta_1 y}\hat{x}$$

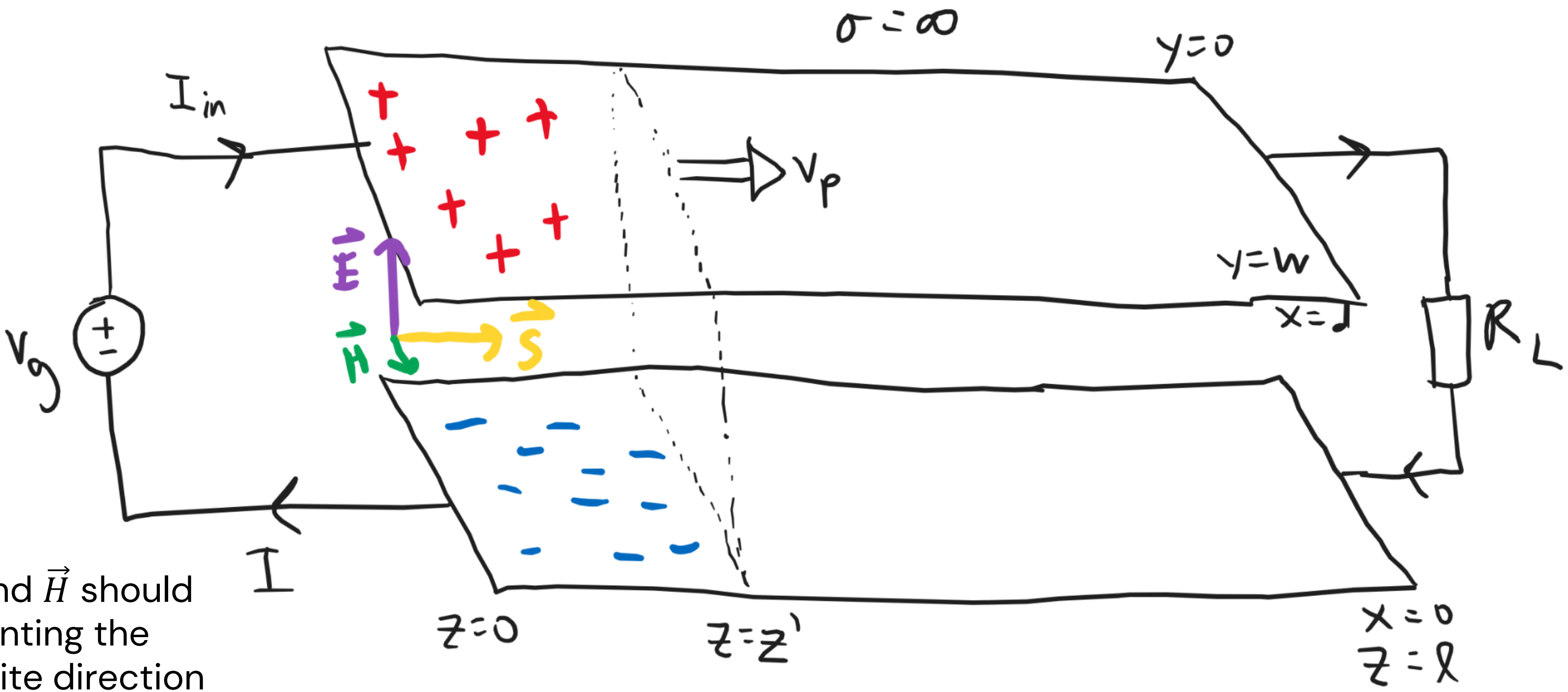
Transmission Lines!

Why do we care?

Transmission Line

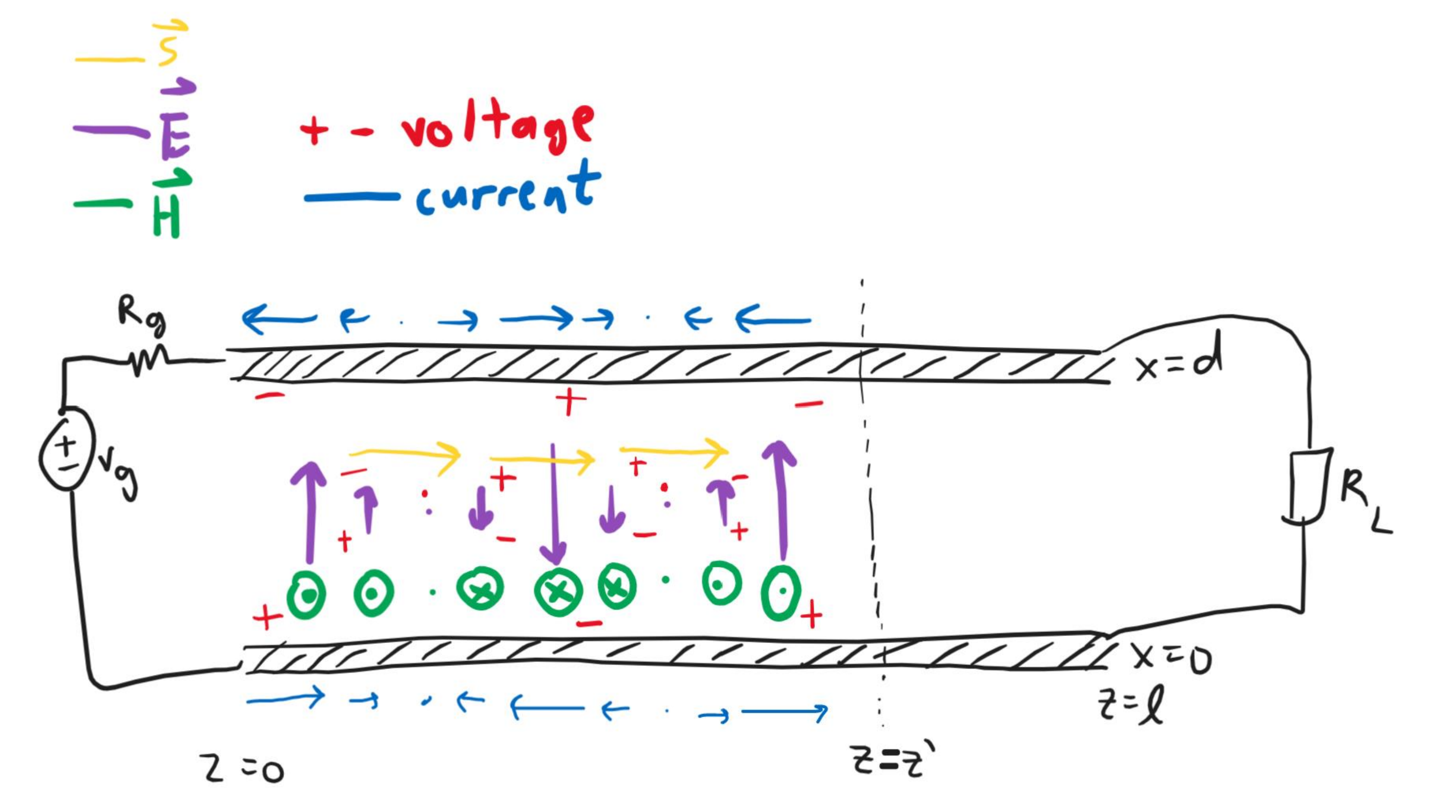


Transmission Line: Parallel Plate Version

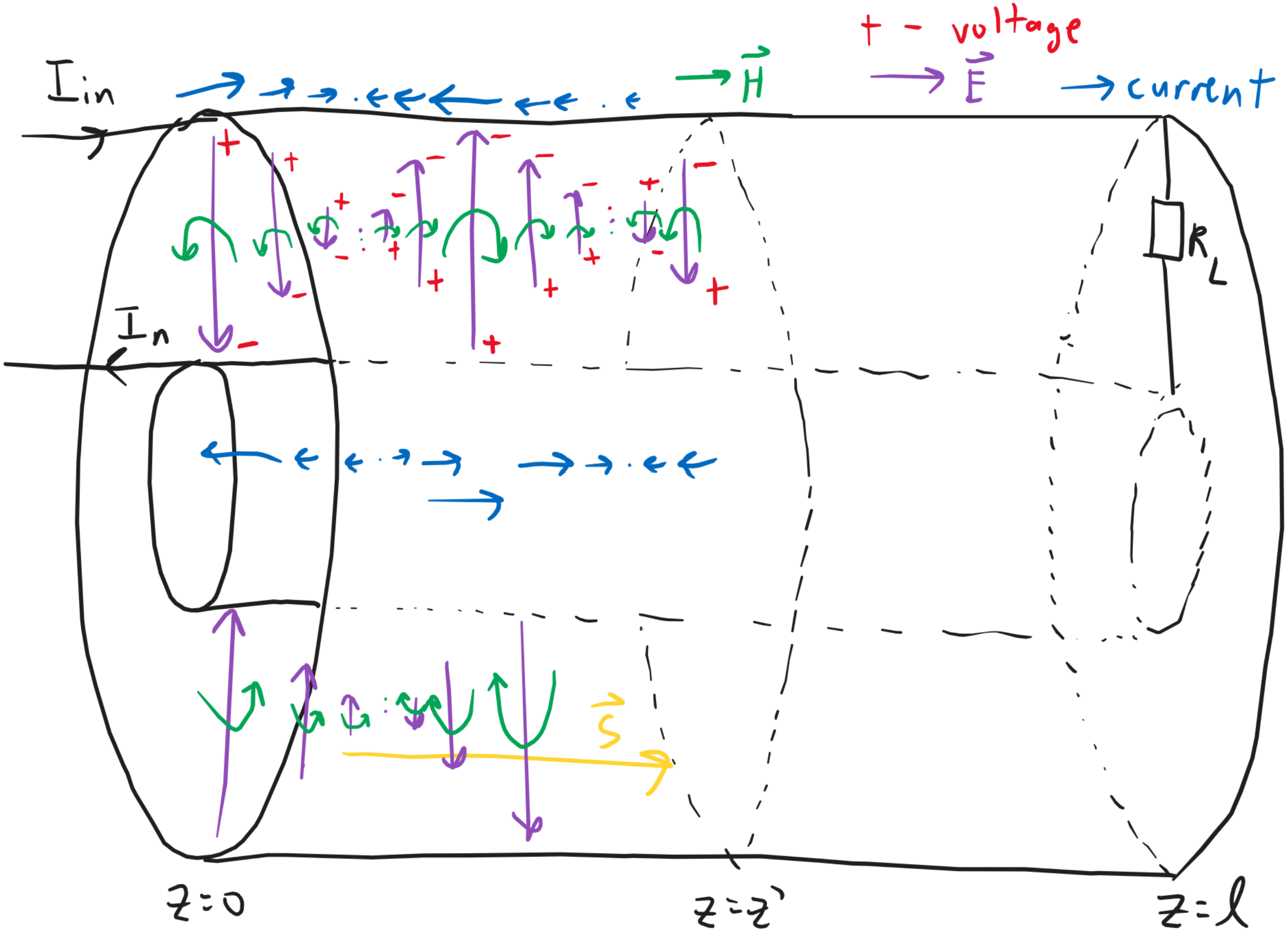


***$\vec{E}$ and $\vec{H}$ should be pointing the opposite direction oops

Transmission Line: Parallel Plate Version



Coax Version:



Basic TL: Time Domain, Not Steady State

Generator injection coefficient: $\tau_g = \frac{Z_0}{R_g + Z_0} \neq 1 + \Gamma_g$

Load reflection coefficient: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Generator reflection coefficient: $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$

Basic TL: Time Domain, Not Steady State

Special cases:

- What if $Z_L = 0$?
- What if $Z_L = \infty$?
- What if $Z_L = Z_0$?

Bounce Diagram Example

Input: $V_g = 50\delta(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

Create a voltage bounce diagram for the first 70ns.

Create a current bounce diagram for the first 70ns.

Plot $V(1\text{m}, t)$ for the first 7us.

Bounce Diagram Example

Input: $V_g = 50\delta(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

Step 1: Calculate all the funny symbols

Bounce Diagram Example

Input: $V_g = 50\delta(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

Step 2: Draw!

Bounce Diagram Example

Input: $V_g = 50\delta(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

Plot $V(1\text{m}, t)$ for the first 70ns.

Bounce Diagram Example

Input: $V_g = 50u(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

Plot $V(1\text{m}, t)$ for the first 70ns.

Bounce Diagram Example

Input: $V_g = 50u(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].

What is the steady-state voltage over the load?
What is the steady-state current through the load?

Bounce Diagrams: General Formulation

This is an LTI system!
Input: $\delta(t)$
Output (at position d): $V(d,t)$
Time to travel down the line: t_0.

$$V(d,t) = \tau_g \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - nt_0) + \tau_g \Gamma_L \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t - \frac{d}{v} - (n+1)t_0)$$

$$I(d,t) = \frac{\tau_g}{Z_0} \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - nt_0) - \frac{\tau_g \Gamma_L}{Z_0} \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t - \frac{d}{v} - (n+1)t_0)$$

For any general input: convolve!

Exam 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Exam 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$\hat{n} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$\hat{n} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s$$

$$\hat{n} \times \left(\vec{M}_1 - \vec{M}_2\right) = \vec{J}_{b,s}$$

Exam 3 equations, in one place

Waves:

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}\hat{z}$$

TLs:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E} + \frac{1}{2}\mu\vec{H}\cdot\vec{H}\right) + \nabla\cdot\vec{S} + \vec{J}\cdot\vec{E} = 0$$

Good luck!