# ECE329: Exam 3 Review Session

November 20th, 2025

# Exam 3 Content

- Poynting Theorem
- Phasors
- TEM Wave Propagation in Material Media
- Polarization
- Standing Waves
- Bounce Diagrams & TLs

# Poynting Vector & Theorem

$\vec{S} = \vec{E} \times \vec{H}$. Units: W/m$^2$. “Instantaneous” power per unit area passing through surface in direction of $\vec{S}$

Energy unit volume balance equation:

$$\underbrace{\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E} + \frac{1}{2}\mu\vec{H}\cdot\vec{H}\right)}_{\text{Total EM Energy density}} + \underbrace{\nabla\cdot\vec{S}}_{\text{Source due to Fields}} + \underbrace{\vec{J}\cdot\vec{E}}_{\text{ohmic loss}} = 0$$

$\bar{J}\cdot\bar{E} < 0 \rightarrow$ source

$\bar{J}\cdot\bar{E} > 0 \rightarrow$ sink

# Phasor Notation

$$\vec{E}(x,t) = A\cos(\omega t - \beta x)\hat{z} = \mathrm{Re}\left\{Ae^{j\omega t}e^{-j\beta x}\hat{z}\right\} \leftrightarrow Ae^{-j\beta x}\hat{z} = \tilde{E}(x)$$

Time domain

Phasor

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

# Time-Averaging

If TEM wave $\vec{E}$ is cosinusoidal,

- Then corresponding $\vec{H}$ is cosinusoidal
- Then phasors $\tilde{E}$ and $\tilde{H}$ exist
- Also, $\vec{S} = \vec{E} \times \vec{H}$ (instantaneous power per unit area) is cosinusoidal squared
- We can write $\vec{S}$ in phasor form too: $\tilde{S} = \tilde{E} \times \tilde{H}^*$
- Time-average of cosinusoidal squared is ½ of the magnitude of the cosinusoidal:

$$\langle \vec{S} \rangle = \frac{1}{T}\int_0^T \vec{S} \cdot dt$$

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{1}{2}\mathrm{Re}\{V I^*\}$$

# Wave Equation in Material Media

$\beta$ [rad/m]

$\alpha$ [Np/m]

Assumptions: $\rho = 0$, $\vec{J} = 0$, i.e. region is a source-free. $\sigma \neq 0$ now!

Wave equation: $\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$. Solutions are TEM waves.
$\vec{E}$ and $\vec{H}$ point **perpendicular** to the direction of travel.

General form of cosinusoidal solution:

$$e^{-\alpha z} \quad e^{-j\beta z}$$

$$\tilde{E} = A e^{-\alpha z} e^{-j\beta z} e^{-j\phi} \hat{x}$$

$$\gamma = \alpha + j\beta$$

$$\vec{E} = A \cos(\omega t - \beta z - \phi) e^{-\alpha z} \hat{x}$$

$$e^{-\gamma z}$$

# Getting H from E

$\eta = |\eta| e^{j\tau}$

$\vec{E} \times \vec{H} = -\hat{y}$

$\hat{x} \times \hat{z} = -\hat{y}$

$$\vec{E} = Ae^{\alpha y}\cos(\omega t + \beta y + \phi)\,\hat{x} \text{ [V/m]}$$

$$\tilde{H} = \frac{A}{|\eta|}\cos(\omega t + \beta y + \phi - \tau)\, e^{\alpha y}\,\hat{z} \left[\frac{A}{m}\right]$$

$$\vec{E} = Ae^{\alpha y}e^{j\beta y}e^{j\phi}\hat{x} \text{ [V/m]}$$

$$\frac{A}{|\eta|} e^{\alpha y} e^{j\beta y} e^{j\phi} e^{-j\tau}\,\hat{z} \left[A/m\right]$$

# Getting E from H

$\eta = |\eta| e^{j\tau}$

$\hat{E} \times \hat{H} = -\hat{y}$

$\hat{z} \times \hat{x}$

$$\vec{H} = Ae^{\alpha y}\cos(\omega t + \beta y + \phi)\,\hat{x} \text{ [A/m]}$$

$\bar{E} = Ae^{\alpha y}|\eta|\cos(\omega t + \beta y + \phi + \tau)\,\hat{z}$ V/m

$$\vec{H} = Ae^{\alpha y}e^{j\beta y}e^{j\phi}\hat{x} \text{ [A/m]}$$

$A|\eta|e^{\alpha y}e^{j\beta y}e^{j\phi}e^{j\tau}\,\hat{z}$ V/m

# EXACT Formulae

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

# APPROXIMATE Formulae

| | Condition | $\beta$ | $\alpha$ | $\lvert\eta\rvert$ | $\tau$ | $\lambda = \frac{2\pi}{\beta}$ | $\delta = \frac{1}{\alpha}$ |
|---|---|---|---|---|---|---|---|
| Perfect dielectric | $\sigma = 0$ | $\omega\sqrt{\epsilon\mu}$ | $0$ | $\sqrt{\frac{\mu}{\epsilon}}$ | $0$ | $\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\infty$ |
| Imperfect dielectric | $\frac{\sigma}{\omega\epsilon} \ll 1$ | $\sim \omega\sqrt{\epsilon\mu}$ | $\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ | $\sim \sqrt{\frac{\mu}{\epsilon}}$ | $\sim \frac{\sigma}{2\omega\epsilon}$ | $\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ |
| Good conductor | $\frac{\sigma}{\omega\epsilon} \gg 1$ | $\sim \sqrt{\pi f\mu\sigma}$ | $\sim \sqrt{\pi f\mu\sigma}$ | $\sqrt{\frac{\omega\mu}{\sigma}}$ | $45°$ | $\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$ | $\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$ |
| Perfect conductor | $\sigma = \infty$ | $\infty$ | $\infty$ | $0$ | - | $0$ | $0$ |

# Wave Polarization

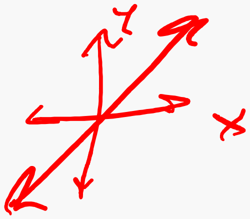
Polarization is how the tip of $\vec{E}$ varies over time.

$A\cos(\omega t - \beta z) + B\cos(\omega t - \beta z - \pi/2)$

**Linear**: In phase $0^\circ$ or $180^\circ$ $\Delta\phi$
**Circular**: 90 degrees out-of-phase with equal magnitude
- Right-Handed
- Left-Handed

**Elliptical**: Anything else

# Wave Polarization




Example of linear polarization: $\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\hat{x}$

Example of right-handed circular polarization (RHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Ahead $\vec{E}$ × Behind $\vec{E}$ = prop. dir → RHCP

$\hat{x} \times \hat{y} = \hat{z}$

Example of left-handed circular polarization (LHCP):

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} - 3\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

RHCP : $\mathrm{Re}\{\tilde{E}\} \times \mathrm{Im}\{-\tilde{E}\}$ = prop. dir

LHCP : $\mathrm{Re}\{\tilde{E}\} \times \mathrm{Im}\{+\tilde{E}\}$ = prop_dir

# Wave Polarization

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + \cos\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{y}$$

Example of elliptical polarization:

$$\vec{\mathrm{E}} = 3\cos(\omega t - \beta z)\,\hat{x} + 3\cos\left(\omega t - \beta z - \frac{\pi}{3}\right)\hat{y}$$

What about this?

$$\vec{\mathrm{E}} = 2\cos(\omega t - \beta z)\,\hat{x} + 4\cos(\omega t - \beta z)\hat{y}$$

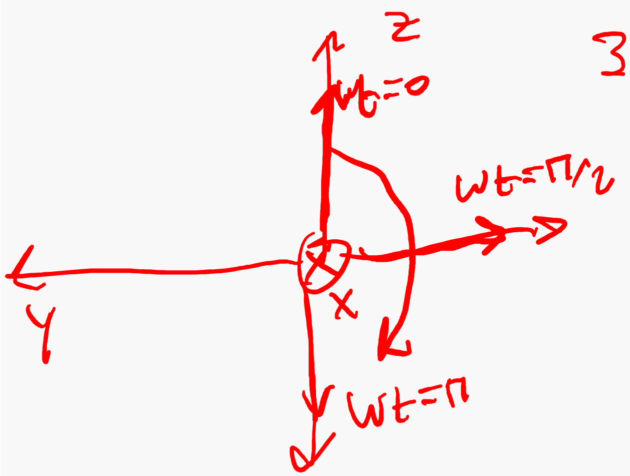
# Wave Polarization Example

Find the polarization of:

$$\vec{E} = \underline{3}\cos(\omega t + \beta x)\,\hat{z} - \underline{3}\cos\left(\omega t + \beta x - \frac{\pi}{2}\right)\hat{y}$$

@ $x=0$ $\quad 3\cos(\omega t)\,\hat{z} - 3\cos\left(\omega t - \frac{\pi}{2}\right)\hat{y}$

$$3\cos(\omega t)\,\hat{z} - 3\sin(\omega t)\,\hat{y} = \vec{E}(0,t)$$




| $\omega t$ | |
|---|---|
| 0 | $3\hat{z}$ |
| $\pi/2$ | $-3\hat{y}$ |
| $\pi$ | $-3\hat{z}$ |

Ahead $\vec{E}$ × Behind $\vec{E}$ = prop. dir

$\hat{z} \times -\hat{y} = +\hat{x}$

# Wave Polarization Example

Find the polarization of:

$$\vec{\mathrm{E}} = 3\cos(\omega t + \beta x)\,\hat{z} - 3\cos\left(\omega t + \beta x + \frac{\pi}{2}\right)\hat{y}$$

RHCP

$-\hat{y} \times \hat{z} = \hat{x}$

?

RHCP

# Wave Reflection & Transmission

TEM wave incident normally on a boundary

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{z}$$

1

2

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

# Reflection & Transmission Coefficients

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_r(x) = E_0 \Gamma e^{+\alpha_1 x} e^{+j\beta_1 x} \hat{y}$$

$$\tilde{H}_r(x) = -\frac{E_0 \Gamma}{\eta_1} e^{+\alpha_1 x} e^{+j\beta_1 x} \hat{z}$$

$$\tilde{E}_t(x) = \tau E_0 e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{y}$$

$$\tilde{H}_t(x) = \frac{\tau E_0}{\eta_2} e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$

$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$

$x > 0$

# Summary

$\tilde{E}_i(0) + \tilde{E}_r(0) = \tilde{E}_t(0)$

$\tilde{H}_i(0) + \tilde{H}_r(0) = \tilde{H}_t(0)$

$$\tilde{E}_i(x) = E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = \frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_t(x) = E_0 \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{y}$$

$$\tilde{H}_t(x) = \frac{E_0}{\eta_2} \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{z}$$

$|\Gamma| \leq 1$

$$\tilde{E}_r(x) = E_0 \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{y}$$

$$\tilde{H}_r(x) = -\frac{E_0}{\eta_1} \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{z}$$

$\sigma \approx \infty$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

# Coefficients

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Check our work:

1. What if $\eta_1 = \eta_2$?

$\Gamma = 0$ $\quad$ $\tau$ ? $\quad$ $\tau = 1$

2. What if $\eta_2 = 0$?

$\eta_2 = 0$

PEC $\rightarrow$ $\Gamma = -1$

$\tau = 0$

# Standing Waves (dielectric to PEC)

$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$\tilde{E}_i(y) = -E_0 e^{-j\beta_1 y}\hat{x}$

$\tilde{E}_r(y) = E_0 e^{j\beta_1 y}\hat{x}$

$$\tilde{E}_{tot} = \tilde{E}_i(y) + \tilde{E}_r(y)$$

$$= -E_0 e^{-j\beta_1 y}\hat{x} + E_0 e^{j\beta_1 y}\hat{x}$$

$$= E_0 \left[e^{j\beta_1 y} - e^{-j\beta_1 y}\right]\hat{x}$$

$\mathrm{Re}\{e^{j\frac{\pi}{2}} e^{j\omega t}\}$

$\mathrm{Re}\{e^{j(\omega t + \frac{\pi}{2})}\}$

$\cos(\omega t + \frac{\pi}{2})$

$$\tilde{E}_{tot}(y) = 2j E_0 \sin(\beta_1 y)$$

$j$

$d$

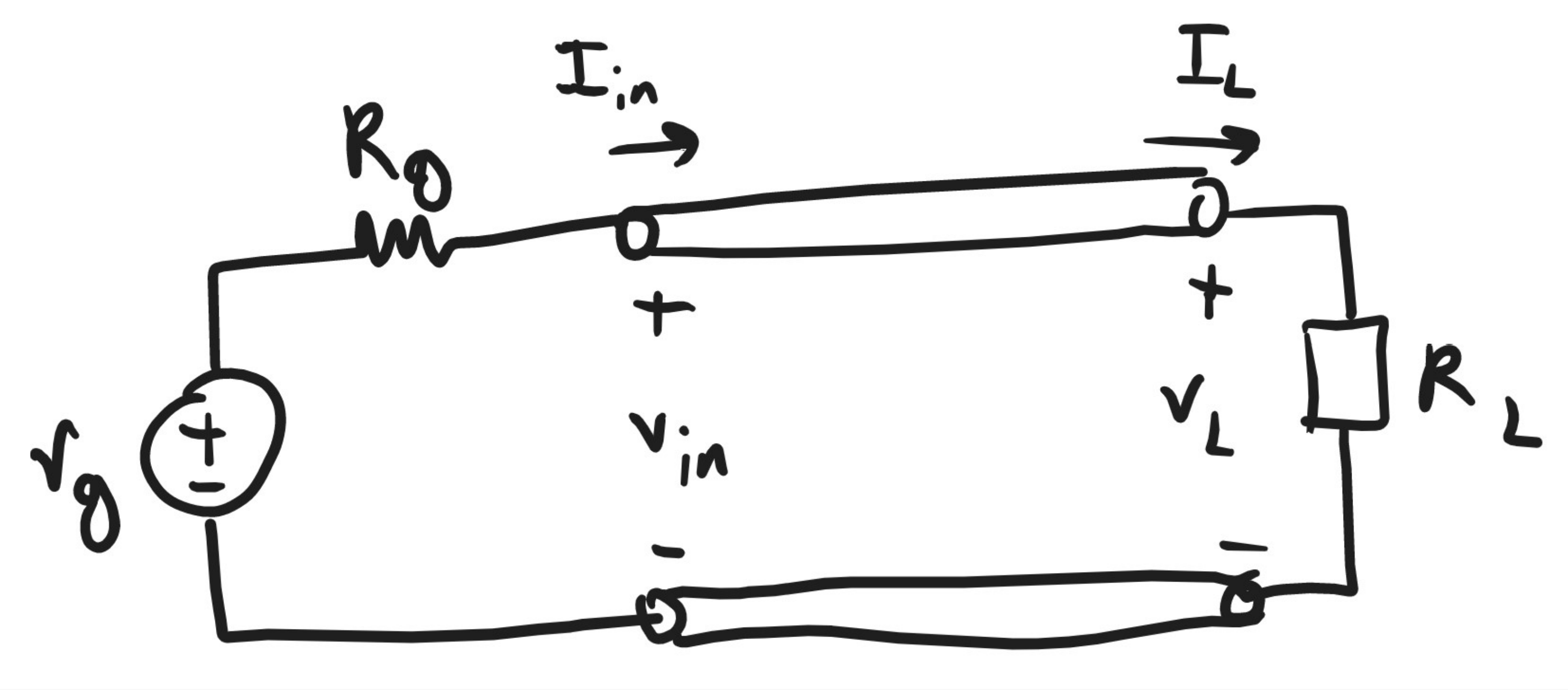
$$\mathrm{Re}\{\tilde{E}_{tot}(y) e^{j\omega t}\} = \mathrm{Re}\{2 E_0 \sin(\beta_1 y)\, e^{j(\omega t + \frac{\pi}{2})}\}$$

$= -2E_0 \sin(\beta_1 y)\sin(\omega t)$

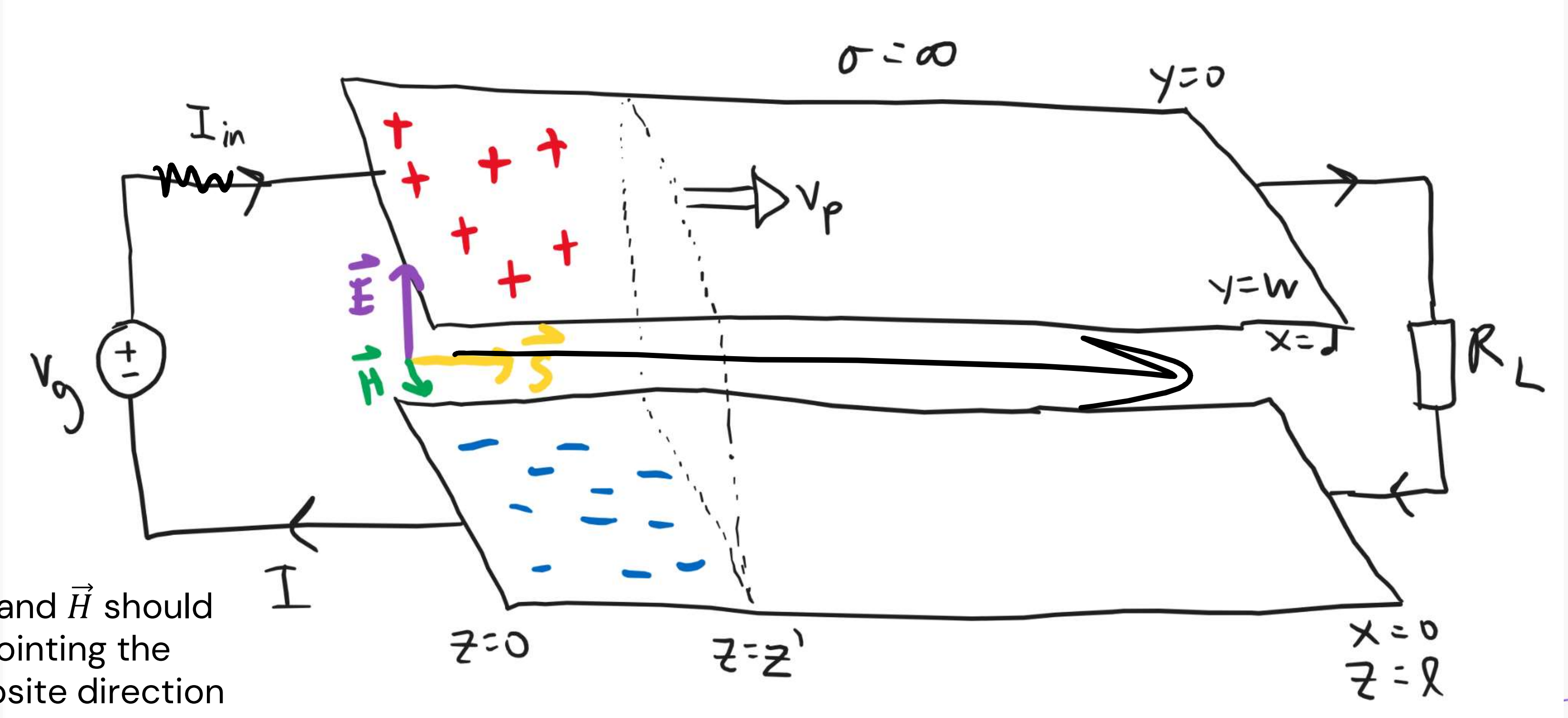
# Transmission Lines!

Why do we care?

# Transmission Line

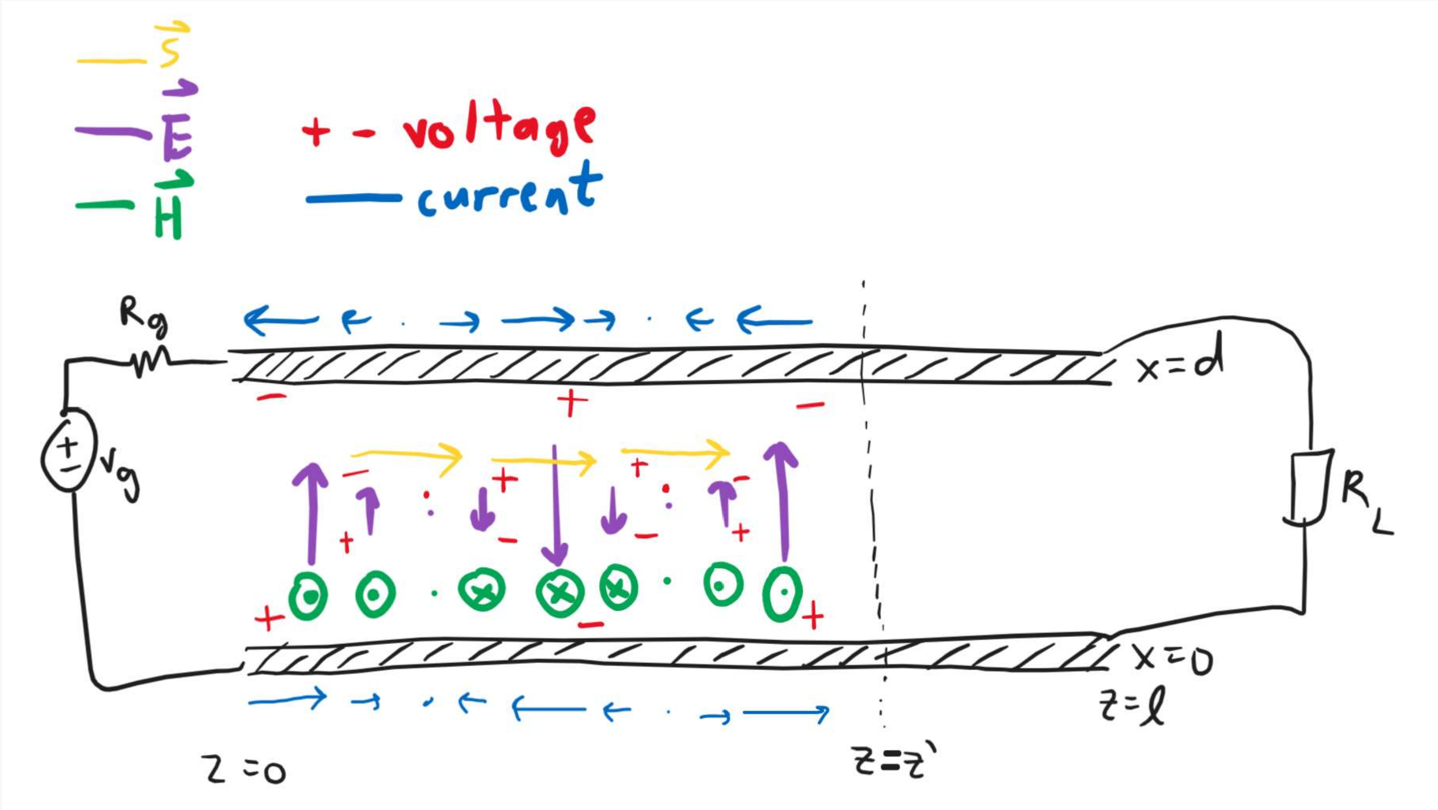


# Transmission Line: Parallel Plate Version

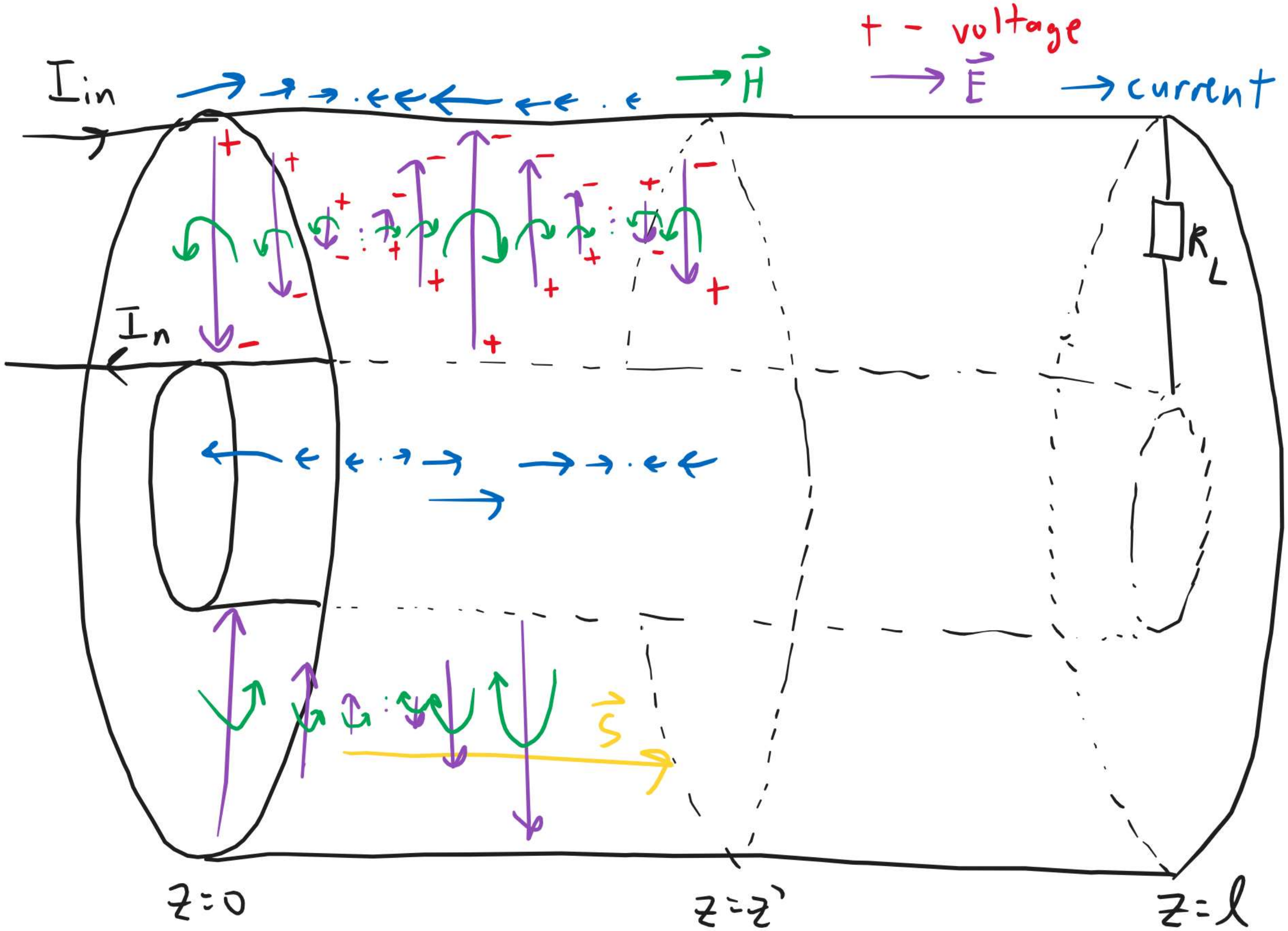



***$\vec{E}$ and $\vec{H}$ should be pointing the opposite direction oops

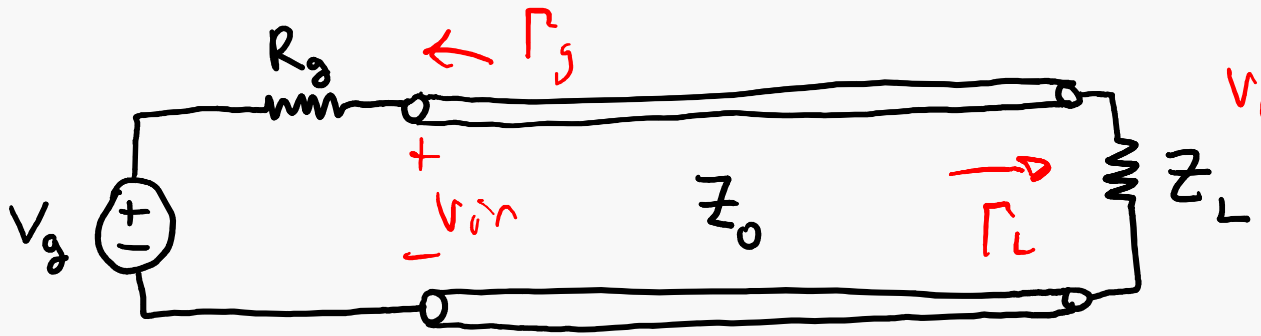
# Transmission Line: Parallel Plate Version



# Coax Version:



# Basic TL: Time Domain, Not Steady State



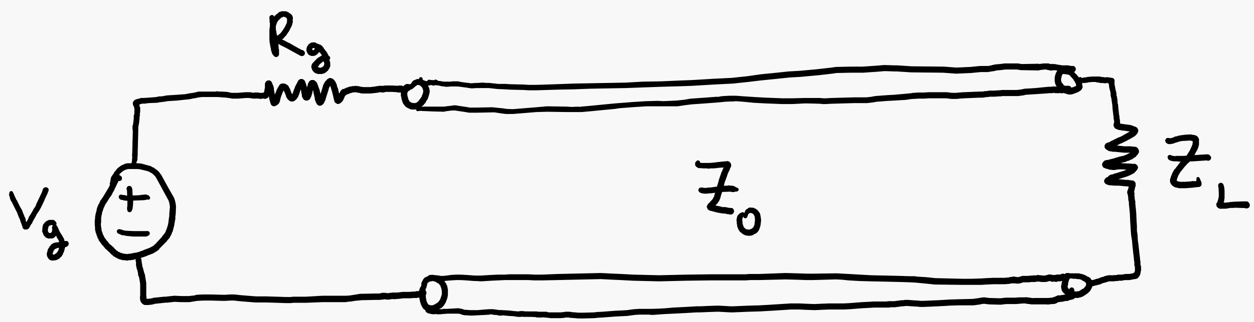

$V_{in} = \tau_g V_g$

Generator injection coefficient: $\tau_g = \boxed{\frac{Z_0}{R_g+Z_0}} \neq 1 + \Gamma_g$

Load reflection coefficient: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Generator reflection coefficient: $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$

# Basic TL: Time Domain, Not Steady State



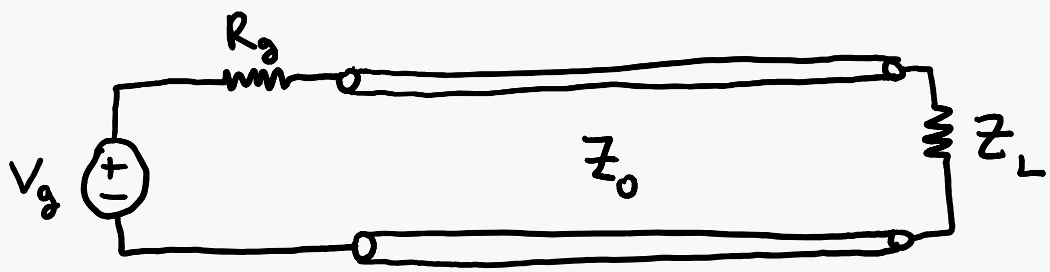

Special cases:

- What if $Z_L = 0$?  $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$  $\Gamma = -1$  Short
- What if $Z_L = \infty$?  $\Gamma = 1$  Open
- What if $Z_L = Z_0$?  $\Gamma = 0$  Matched load

# Bounce Diagram Example

Input: $V_g = 50\delta(t)$ [V]
$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$
$v = \frac{2}{3}c$, TL length = 4 [m].

Create a voltage bounce diagram for the first 70ns.
Create a current bounce diagram for the first 70ns.
Plot $V(1\text{m}, t)$ for the first 7us.



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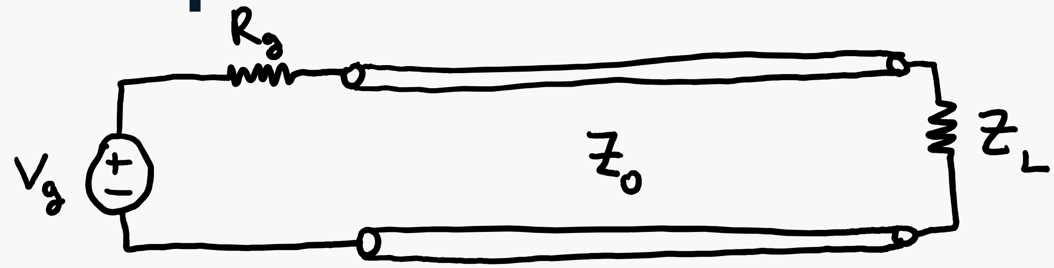



Step 1: Calculate all the funny symbols

$\tau_g, \Gamma_g, \Gamma_L \rightarrow \tau_g = \frac{Z_0}{R_g+Z_0} = \frac{2}{3}, \quad \Gamma_g = \frac{R_g-Z_0}{R_g+Z_0} = \frac{50-100}{50+100} = -\frac{1}{3}$

$\Gamma_L = \frac{Z_L-Z_0}{Z_L+Z_0} = \frac{25-100}{25+100} = \frac{-75}{125} = -3/5$

$\frac{l}{v} = \frac{4m}{2/3\, c} = 20ns \leftarrow$ one-way prop-delay

# Bounce Diagram Example

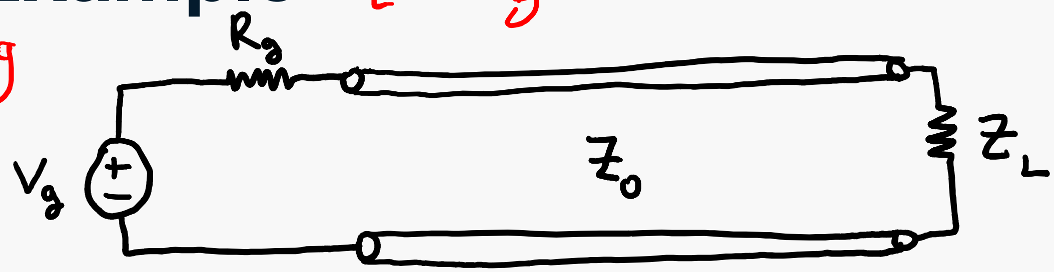
$\tau_g = 2/3$   $\Gamma_g = -1/3$ , $\frac{l}{v} = 20ns$

$\Gamma_L = -3/5$

Input: $V_g = 50\delta(t)$ [V]   $50 \cdot \tau_g$

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

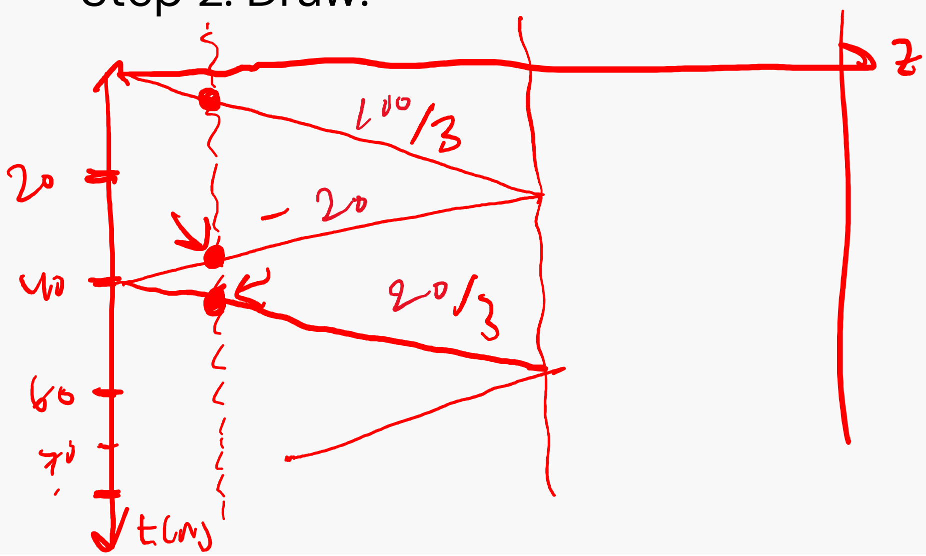
$v = \frac{2}{3}c$, TL length = 4 [m].




$V(1m, t)$

70ns

Step 2: Draw!

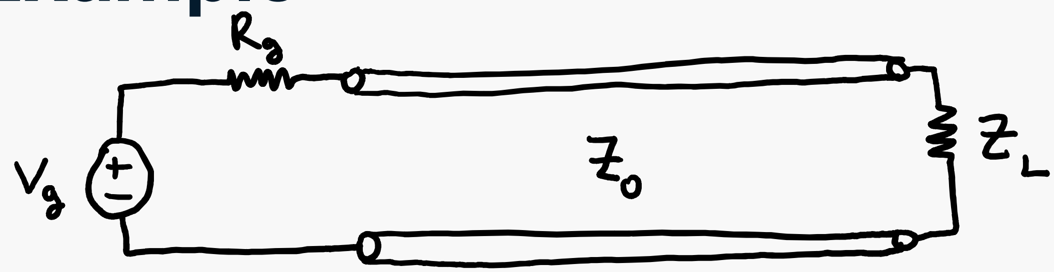


# Bounce Diagram Example

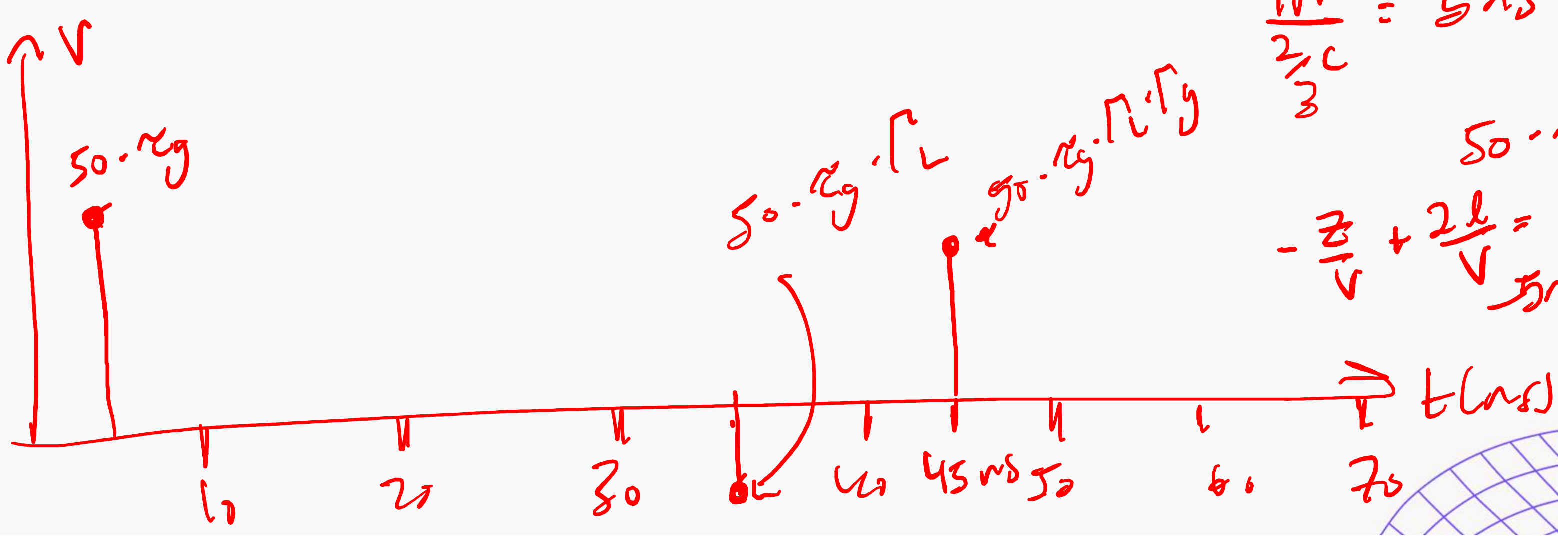
Input: $V_g = 50\delta(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = \frac{2}{3}c$, TL length = 4 [m].




Plot $V(1\text{m}, t)$ for the first 70ns.



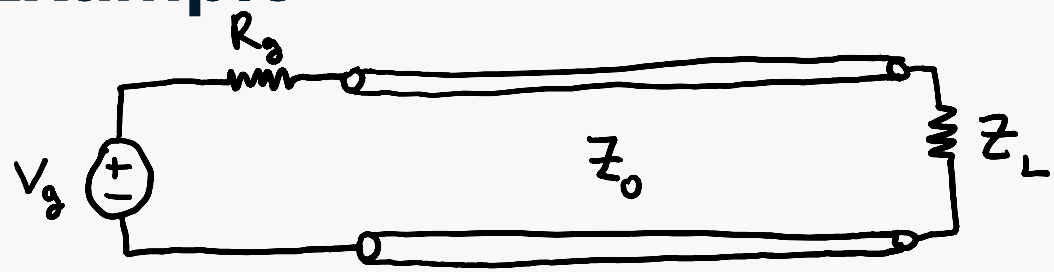

$\frac{1\text{m}}{\frac{2}{3}c} = 5\text{ns}$

$50 \cdot \tau_g$

$-\frac{z}{v} + \frac{2l}{v} = 5\text{ns} + 40\text{ns}$

# Bounce Diagram Example

Input: $V_g = 50u(t)$ [V]
$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$
$v = \frac{2}{3}c$, TL length = 4 [m].

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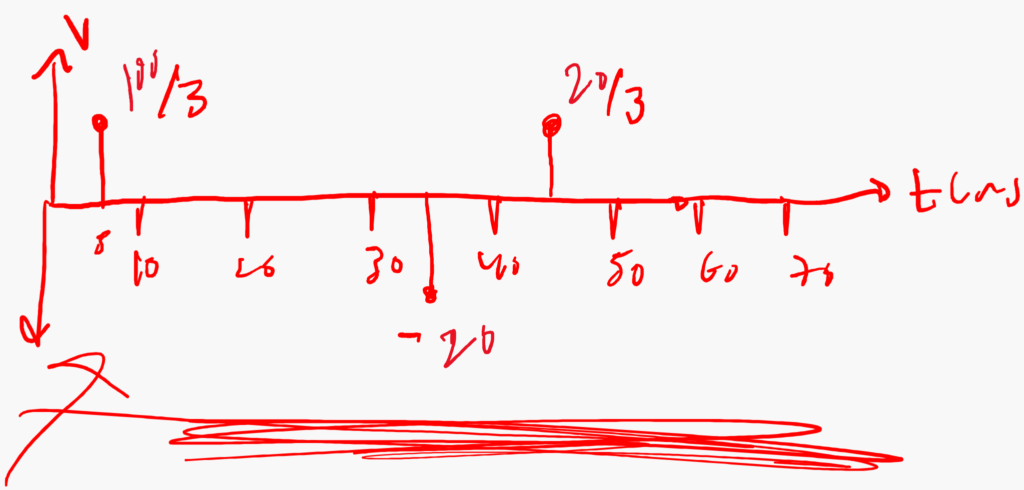


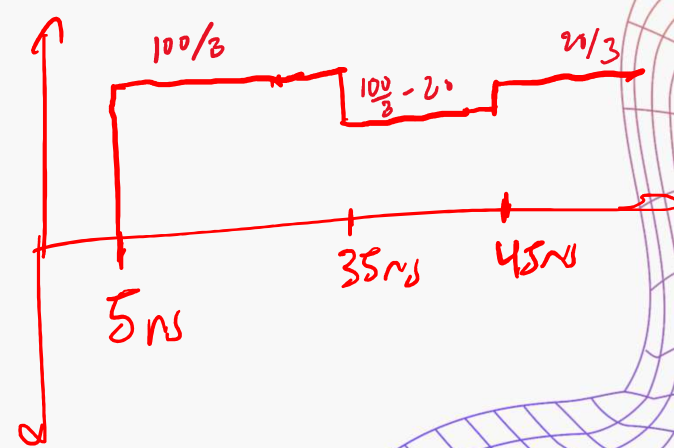

# Bounce Diagram Example

Input: $V_g = 50u(t)$ [V]

$Z_L = 25\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

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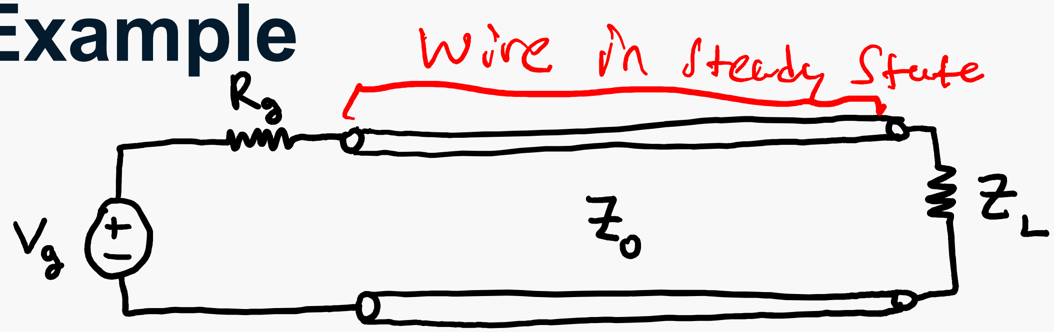



What is the steady-state voltage over the load?

What is the steady-state current through the load?

$$V_{ss,L} = 50 \cdot \frac{25}{50+25} = \frac{25}{75} = \frac{50}{3}\ \text{V}$$

$$I_{ss,L} = \frac{V_{ss,L}}{R_L} = \frac{50/3}{25} = \frac{2}{3}\ \text{A}$$

# Bounce Diagrams: General Formulation

This is an LTI system!
Input: $\delta(t)$
Output (at position $d$): $V(d,t)$
Time to travel down the line: $t_0$.

$\tau_g \delta(t - \frac{z}{v})$
$z = 1m$

1m

5ns

$$V(d,t) = \tau_g \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - nt_0) + \tau_g \Gamma_L \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - (n+1)t_0)$$

$$I(d,t) = \frac{\tau_g}{Z_0} \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - nt_0) - \frac{\tau_g \Gamma_L}{Z_0} \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - (n+1)t_0)$$

For any general input: convolve!

# Exam 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\hat{n}\cdot\left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

# Exam 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$\hat{n} \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$\hat{n} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s$$

$$\hat{n} \times \left(\vec{M}_1 - \vec{M}_2\right) = \vec{J}_{b,s}$$

# Exam 3 equations, in one place

***Waves***:

| | Condition | $\beta$ | $\alpha$ | $\lvert\eta\rvert$ | $\tau$ | $\lambda = \frac{2\pi}{\beta}$ | $\delta = \frac{1}{\alpha}$ |
|---|---|---|---|---|---|---|---|
| Perfect dielectric | $\sigma = 0$ | $\omega\sqrt{\epsilon\mu}$ | 0 | $\sqrt{\frac{\mu}{\epsilon}}$ | 0 | $\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\infty$ |
| Imperfect dielectric | $\frac{\sigma}{\omega\epsilon} \ll 1$ | $\sim \omega\sqrt{\epsilon\mu}$ | $\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ | $\sim \sqrt{\frac{\mu}{\epsilon}}$ | $\sim \frac{\sigma}{2\omega\epsilon}$ | $\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ |
| Good conductor | $\frac{\sigma}{\omega\epsilon} \gg 1$ | $\sim \sqrt{\pi f \mu\sigma}$ | $\sim \sqrt{\pi f \mu\sigma}$ | $\sqrt{\frac{\omega\mu}{\sigma}}$ | 45° | $\sim \frac{2\pi}{\sqrt{\pi f \mu\sigma}}$ | $\sim \frac{1}{\sqrt{\pi f \mu\sigma}}$ |
| Perfect conductor | $\sigma = \infty$ | $\infty$ | $\infty$ | 0 | - | 0 | 0 |

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow A e^{-j\beta x}\hat{z}$$

***TLs***:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\gamma\eta = j\omega\mu$$

$$\frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

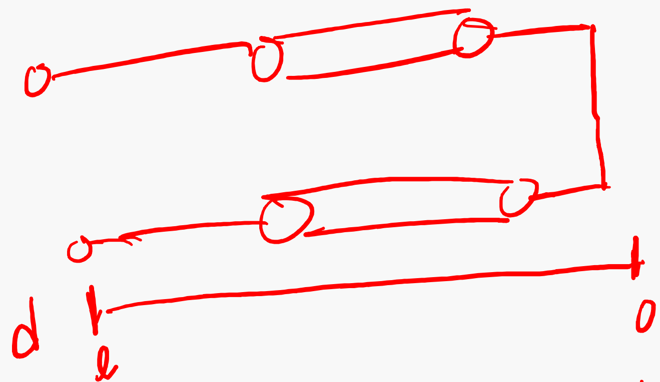
$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E}\cdot\vec{E} + \frac{1}{2}\mu\vec{H}\cdot\vec{H}\right) + \nabla\cdot\vec{S} + \vec{J}\cdot\vec{E} = 0$$

# Good luck!




$$\tilde{I}(0) = \text{Max}$$
$$\tilde{I}(l) = 0$$
$$\tilde{V}(0) = 0$$
$$\tilde{V}(l) = \text{Max}$$

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)} = jZ_0 \tan(\beta d)$$

$$V(d) = V^+ e^{+j\beta d} + V^- e^{-j\beta d}$$

$$I(d) = \frac{V^+}{Z_0} e^{+j\beta d} - \frac{V^-}{Z_0} e^{-j\beta d}$$

$$V^- = \Gamma V^+ \Rightarrow V^- = -V^+$$

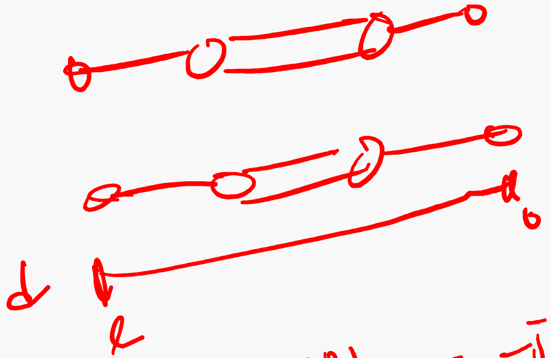
$$V(d) = V^+ e^{j\beta d} - V^+ e^{-j\beta d}$$

$$V(d) = V^+ \left(e^{j\beta d} - e^{-j\beta d}\right)$$

$$\tilde{V}(d) = 2jV^+ \sin(\beta d)$$

$$\tilde{I}(d) = \frac{2V^+}{Z_0} \cos(\beta d)$$

$$I(d) = \frac{V^+}{Z_0}\left(e^{j\beta d} + e^{-j\beta d}\right)$$



$$\Gamma = 1$$

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$= V^+ \left(e^{j\beta d} + e^{-j\beta d}\right)$$

$$= 2V^+ \cos(\beta d)$$

$$I(d) = \frac{V^+}{Z_0}\left[e^{j\beta d} - e^{-j\beta d}\right] = \frac{2jV^+}{Z_0}\sin(\beta d)$$

$$\frac{V(d)}{I(d)} = \frac{2V^+ \cos(\beta d)}{\frac{2jV^+}{Z_0}\sin(\beta d)} = -jZ_0 \cot(\beta d)$$