- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

Reading Assignment: Kudeki: Lectures 17-20

1. Verify that vector identity

$$\mathbf{H}\cdot\nabla\times\mathbf{E}-\mathbf{E}\cdot\nabla\times\mathbf{H}=\nabla\cdot(\mathbf{E}\times\mathbf{H})$$

holds for $\mathbf{H}=4\hat{x}e^{-\alpha z}$ and $\mathbf{E}=2\hat{y}e^{-\alpha z}$ by expanding both sides of the identity. Treat α as a real constant.

You should download the table of vector identities from ECE 329 web site and examine the list to familiarize yourself with the listed identities — they are widely employed in electromagnetics as well as in other branches of engineering such as fluid dynamics.

2.

a) For current density $\mathbf{J}=(5z^2\,\hat{x}+4x^3y\,\hat{y}+3z(y-y_o)^2\,\hat{z})$ A/m^2, which is time independent, find the charge density $\rho(0,t)$ at the origin (0,0,0) as a function of time t, if $\rho=0$ at that location and time $t=0$, $y_o=2$ m, and coordinates x, y, and z are specified in meter units. **Hint:** use the continuity equation

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0.$$

b) In part (a), deduce the physical units of the coefficients 5, 4, and 3 used in J_x, J_y, and J_z specifications, respectively, by applying dimensional analysis.

3.

a) Show that in a homogeneous conductor where $\mathbf{J}=\sigma\mathbf{E}$, Gauss's law $\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_o}$ and the continuity equation $\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0$ can be used together to derive a differential equation

$$\frac{\partial\rho}{\partial t}+\frac{\sigma}{\epsilon_o}\rho=0$$

for the charge density ρ.

b) Find the solution of the differential equation above for $t>0$ if at $t=0$ the charge density is $\rho(x,y,z,0)=\sin(500x)$ C/m^3 over all space.

c) According to the solution found in part (b), how long would it take for ρ to reduce to $0.05\sin(500x)$ C/m^3? Assume that $\sigma=1.0\times10^7$ S/m.

d) Discuss the energetics of the process examined above: Specifically, state whether the energy per unit volume is zero or non zero at $t=0$ and as $t\to\infty$ and state what happens to any energy stored in the quasistatic field at $t=0$.

4.

a) If

$$\mathbf{E}=\cos(\omega t-\beta x)\,\hat{y}\,\frac{\text{V}}{\text{m}},$$

$\frac{\omega}{\beta}=c$, and $\mu=\mu_o$, find the corresponding $\mathbf{H}$ by using Faraday's law

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}.$$

Hint: find a suitable time varying anti-derivative for $\nabla\times\mathbf{E}$.

b) If

$$\mathbf{H} = \cos(\omega t + \beta y)\,\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}},$$

$\sigma = 0$, $\frac{\omega}{\beta} = \frac{2}{3}c$ and $\epsilon = 2.25\epsilon_o$, find the corresponding $\mathbf{E}$ by using Ampere's law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

in which $\mathbf{J} = \sigma \mathbf{E}$ and $\mathbf{D} = \epsilon \mathbf{E}$.