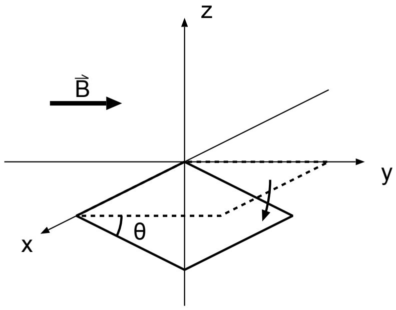
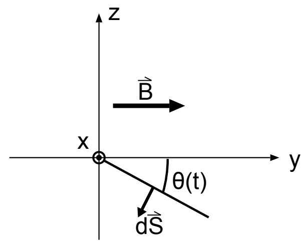
ECE 329 **Homework 6** **Due: (Extended to) Monday, October 6, 2025, 4:59:00PM**

- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

Reading Assignment: Kudeki: Lectures 14-17

1. A square loop of wire of some finite resistance R and 4 cm^2 surface area is located within a region of constant magnetic field $\mathbf{B} = 8\hat{y}$ Wb/m^2 as illustrated in the following diagrams (perspective and side views are shown).





   a) What is the magnetic flux Ψ through the loop when the orientation angle of the loop is $\theta = 0^o$? When the orientation angle of the loop is $\theta = 90^o$? In your flux calculation make use of **dS** orientation shown in the diagram on the right.

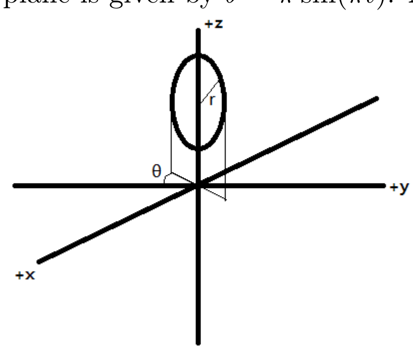
   b) What is the flux Ψ as a function of angle θ (using the same sign convention as in part a)?

   c) Assuming that angle θ is time varying at a rate of $\frac{d\theta}{dt} = \pi \frac{\text{rad}}{\text{s}}$, and $d\mathbf{S}$ is pointing in the $-\hat{z}$ at time $t = 0$s. What is the emf $\mathcal{E}$ around the loop at $t = 0.25$s?

   d) In what direction will a positive induced current flow around the loop at the same instant? You may draw a picture to explain your answer. Be sure to justify your answer.

   e) What is the emf $\mathcal{E}$ derived from using the **opposite dS** orientation to that shown in the figure? In what direction will a postive induced current flow around the loop in this case?

2. On Planet X, a metal hoop of radius $r = \frac{1}{2}$m initially lies entirely on the yz plane and is centered around the z-axis. At the location of the hoop, the magnetic field of Planet X is given by $\mathbf{B} = B_0\hat{x}\text{Wb/m}^2$. At $t = 0$, the loop begins to rotate about the z-axis, where the angle made with the yz plane is given by $\theta = \pi\sin(\pi t)$. At $t = 0$, $\hat{n}$ for the loop points in the $\hat{x}$ direction.



   a) If the emf induced in the loop is 10^{-4}V at $t = 0.3$s, what is the value of B_0?

   b) A study finds that Planet X's magnetic field is actually time varying and is given by $\mathbf{B} = B_0\cos(\frac{\pi}{2}t)\hat{x}\text{Wb/m}^2$. Write the expression for the magnetic flux through the loop for $t > 0$ as a function of time.

   c) What is the induced emf in the loop at $t = 4$ for the magnetic field given in part (b)?

3. A conducting wire loop of radius $r = 1$ m is moved with velocity $\mathbf{v} = 2\hat{x}$ m/s in a region where the background magnetostatic field is described by

$$\mathbf{B}(x, y, z) = \hat{z}\, 25 \times 10^{-6}(1 - x/L)\,\mathrm{T},$$

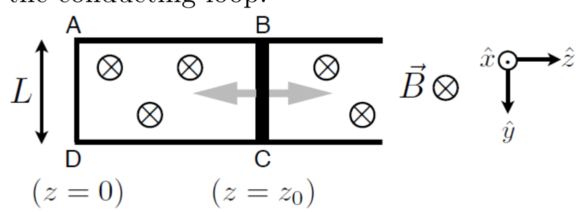
where $L = 1000$ m. The center of the loop coincides with the origin $(x, y, z) = (0, 0, 0)$ at $t = 0$ and the plane of the loop coincides with $z = 0$ plane.

a) Obtain an expression for the induced emf $\mathcal{E}(t)$ of the loop in motion for $t > 0$. Since $r \ll L$, the magnetic field across the loop can be considered nearly constant at each instant in time.

b) What is the magnitude of the loop current as a function of time for $t > 0$ if the loop resistance is $2\,\Omega$?

Interesting facts: The strength of Earth's magnetic field is just about 25×10^{-6} T at equatorial latitudes. However, the scale length L associated with the spatial variation of Earth's magnetic field is much longer than 1000 m.

4. As shown in the diagram below, a pair of conducting rails separated by a distance L is connected at $z = 0$ to a fixed conducting rod (AD) and at $z = z_0$ to a conducting armature (BC) that can slide along the rail in the $\pm\hat{z}$ direction. A constant magnetic field $\mathbf{B} = -B_0\,\hat{x}$ exists in the region, which is shown pointing down into the page in the diagram. The armature is mechanically pulled in the $+\hat{z}$ direction at a constant velocity $\mathbf{v} = v_0\hat{z}$ m/s from its starting position at $z = z_0$, and the changing magnetic flux through the loop ABCD induces an electromotive force $\mathcal{E}$ and thus a current I_0 around the conducting loop.



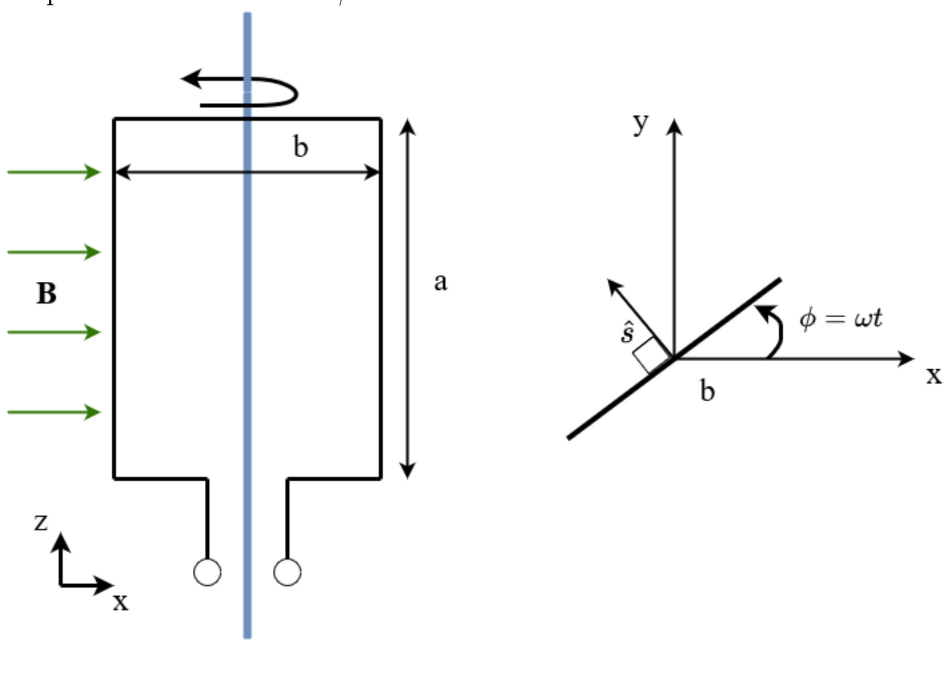
a) What is the emf $\mathcal{E}$ induced in the loop ABCD?

b) What is the magnitude and direction of the induced current I_0 in terms of the resistance R of the conducting loop ABCD?

c) What is the magnitude and direction of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ that is exerted on the armature by the magnetic field $\mathbf{B}$? **Hint**: express your answer in terms of the induced current I_0.

d) Now consider that the armature is no longer moved mechanically (though it is still free to move in response to Lorentz forces) and that a projectile of mass M is attached to it. A constant current of magnitude I' is introduced into the loop at point A such that it flows along the contour ABCD. The magnetic field generated by this current loop is negligibly small compared to the background magnetic field $\mathbf{B}$, and the mass of the armature is negligibly small compared to M. What is the magnitude and direction of the acceleration $\mathbf{a}$ of the projectile? **Hint**: $\mathbf{F} = M\mathbf{a}$.

5. A rectangular conducting loop of sidelengths a and b is mounted on a vertical shaft in the $\hat{z}$ direction, with a uniform magnetic field pointing in the $\hat{x}$ direction ($\mathbf{B} = B\hat{x}$), as shown in the figure above. The loop, denoted C, encloses an area denoted S, with unit normal vector $\hat{s}$ and infinitesimal area vector $d\mathbf{S} = \hat{s}dS$. Figure (A) depicts a head-on view of the system when $\phi = 0$, and figure (B) depicts a top-down view to define ϕ.



(A) (B)

a) Find the unit normal vector $\hat{s}$ as a function of ϕ, the angle between $\hat{s}$ and $\hat{x}$. **Hint:** notice that $\hat{s}$ remains perpendicular to the surface for any ϕ, and that $\hat{s}(\phi = 0) = \hat{y}$.

b) Using this expression, calculate the magnetic flux through the loop,

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

for an arbitrary angle $\phi \in [0, 2\pi)$.

c) The loop now spins counterclockwise at a rate of ω [rad/s], meaning its angle relative to the x-axis is $\phi(t) = \omega t$. Using your expression from part (b), find the magnetic flux through the loop as a function of time. Using Faraday's law, calculate the induced emf on the loop.

d) Consider the loop having an internal resistance R. Find and plot the current flowing through the loop for $a = b = R = \omega = B = 1$, on a range $t \in [0, 2\pi)$ [s].