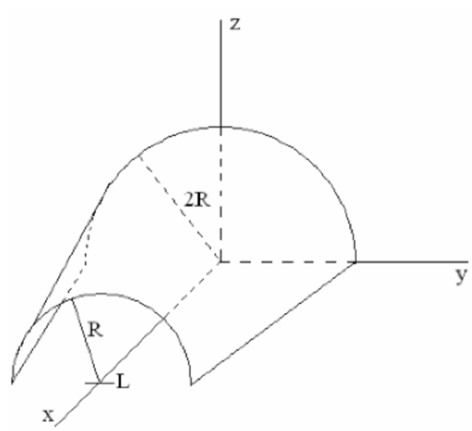
- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

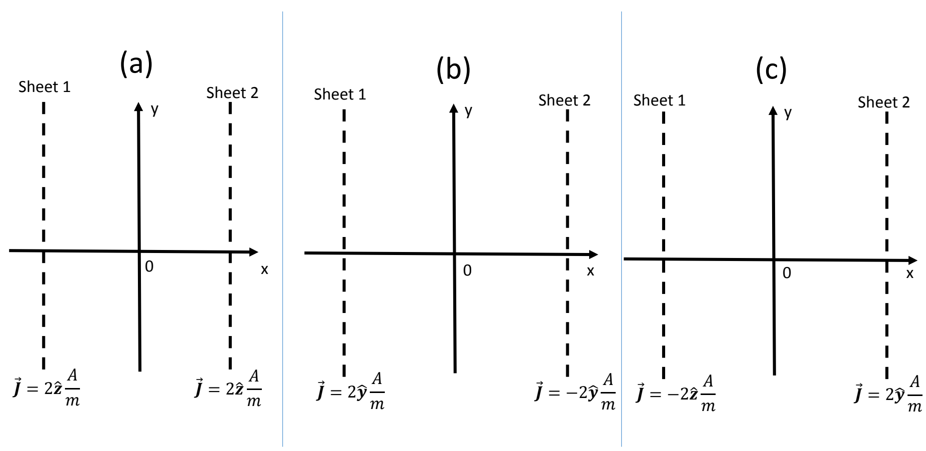
Reading Assignment: Kudeki: Lectures 12-14

1. Gauss' Law for **B** states that $\oint_S \mathbf{B}\cdot\mathbf{dS} = 0$ over any closed surface S enclosing a volume V. Given that $\mathbf{B} = B_0(-3\hat{x} + 2\hat{y} - \frac{\pi R}{L}\hat{z})$ Wb/m^2, determine the magnetic flux through the partial cone surface shown in the following figure:



2. An infinite current sheet with a uniform current density $\mathbf{J} = J_s\hat{y}\ \frac{\mathrm{A}}{\mathrm{m}}$ produces magnetostatic fields **B** with $\frac{\mu_o J_s}{2}$ magnitude on both sides of the sheet and with opposing directions in consistency with the right-hand-rule and the Biot-Savart law.

   Determine the **magnetic field intensity** $\mathbf{H} = \frac{\mathbf{B}}{\mu_o}$ at origin O in the following diagrams due to a pair of current sheets with specified **J** vectors.



3. Consider a hollowed out cylinder centered on the z-axis with inner radius a and outer radius b such that it is described by $a < r < b$. The hollow cylinder conducts a uniform current density of $\mathbf{J} = -J_0\hat{z}\frac{A}{m^2}$ in the region $a < r < b$. Outside this region, that is for $r > b$ and $r < a$, the charge and current densities are zero.

   a) Using the integral form of Ampere's Law, find $\mathbf{B}$ everywhere (you can simplify your answer using $I = J_0 A$).

   b) For $r > b$, what does the magnetic field $\mathbf{B}$ produced by a hollow cylinder look like? (Hint: See Lecture 12 online, this is the magnetic analog to a hollow sphere of charge).

   c) Now assume the hollow cylinder conducts a non-uniform current density of $\mathbf{J} = \frac{-J_0}{r}\hat{z}$. Find $\mathbf{B}$ everywhere.

   d) Prove that Gauss' Law for the magnetic field in (c) is satisfied. In cylindrical coordinates,

$$\nabla \cdot \mathbf{F} = \frac{1}{r}\frac{\partial(rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}.$$

4. Consider an infinite slab (extending in y and x directions) of a finite width $W = 4$ m described by $-3 < z < 1$. The slab is electrically neutral but it conducts a uniform current density of $\mathbf{J} = \mathbf{2\hat{y}\frac{A}{m^2}}$ (meaning that it contains equal densities of positive and negative charge carriers moving in opposite directions parallel to $\hat{y}$). Outside the slab, that is for $z > 1$ and $z < -3$, the charge and current densities are zero.

   a) Using the right-hand-rule and Biot-Savart law, discuss why the current slab should generate equal and opposite directed magnetic fields in $\pm\hat{x}$ directions in front of and behind the plane of symmetry of the slab.

   b) Based on part (a), what is $\mathbf{B}$ on the $z = -1$m plane? Briefly explain the reasoning behind your answer.

   c) Next, make use of the integral form of Ampere's law and the deductions of parts (a) and (b), to find $B_x(z)$ in the regions outside the slab. Hint: make use of a shifted coordinate system with its origin at the center of the slab.

   d) Use Ampere's Law to find $B_x(z)$ at a distance z within the current slab.

   e) Plot B_x as a function of x over $-5 < z < 3$. Be sure to label all relevant values of B_x and z.

5. **Bonus Problem**: In the parallel-plate magnetron a uniform electric field $\mathbf{E} = -(V_0/d)\hat{z}$ is established between two parallel plates together with a uniform magnetic field $\mathbf{B} = -B\hat{y}$. The lower plate, the cathode, is located on the$z = 0$ plane and held at ground potential ($V = 0$). The upper plate, the anode, is maintained at a potential V_0 and is located at $z = d$. Electrons with charge $-|e|$ and mass m are released with neligible velocity from the lower plate and are accelerated toward the upper plate by the force due to the electric field $\mathbf{F_e} = -|\mathbf{e}|\mathbf{E}$. Since the magnetic force $\mathbf{F}_m = -|e|\mathbf{u} \times \mathbf{B}$ is always at right angles to the velocity $\mathbf{u}$, the electrons describe a curved path. Find the minimum potential difference V_0 between the plates needed for the electrons to reach the upper plate. This potential is known as the Hull cut-off in magnetron design.
   Hint: Use conservation of energy and the fact that the magnetic force does not contribute to the kinetic energy of the electrons. Ignore the effects of gravity.