- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

Reading Assignment: Kudeki: Lectures 9-11

1. Consider two infinite, plane parallel, perfectly conducting plates at $z = 0$ and $z = z_0 > 0$, which hold equal and opposite surface charge densities and are kept at potentials $V = 0$ and $V = V_p > 0$, respectively. The region between the plates is filled with two slabs of perfect dielectric materials having permittivities ϵ_1 for $0 < z < d$ (region 1) and ϵ_2 for $d < z < z_0$ (region 2).

    a) Find the general solution for the electric potentials, $V = V(z)$, (in terms of V_p, d, z_0, ϵ_1, and ϵ_2) in the two regions by solving Laplace's equation piecewise and enforcing the continuity of $V(z)$ at $z = d$. **Hint:** you will also need to use the fact that there is no surface charge accumulation at a boundary between two perfect dielectrics.

    b) Given that $z_0 = 4d = 2\,\text{m}$, $\epsilon_1 = 3\epsilon_0$, $\epsilon_2 = \epsilon_0$, and $\mathbf{E}(0 < z < d) = -5\hat{z}\,\frac{\text{V}}{\text{m}}$, what is the electrostatic potential V_p on the conductor plate at $z = z_0$?

    c) Given the parameters in part (b) above, what is the surface charge density ρ_s on the plate at $z = z_0$?

    d) Does $V(z)$ determined in part (a) satisfy Laplace's equation in the region $0 < z < z_0$ m? Explain your answer.

    e) Determine the capacitance C of the structure described above if the parallel plates at $z = 0$ and $z = z_0$ m were constrained to have finite areas $A = W^2$ facing one another (where $W \gg z_0$ so that fringing effects can be neglected). In this calculation ignore the fringing fields, and express C as a function of ϵ_1, ϵ_2, d, z_0 and A.

2. Consider two conducting plates positioned on $z = 0$ and 10 m surfaces. The plates are grounded and both have zero potential. In between the plates, on $z = 4\,\text{m}$ surface, there is a uniform and static surface charge of $8\epsilon_o\,\frac{\text{C}}{\text{m}^2}$. The permittivity of the region $z < 4$ m is ϵ_o, whereas it is $3\epsilon_o$ in the region $z > 4$ m. Determine the surface charge densities at $z = 0$ and 10 m.

    Hint: Let V_o denote the electrostatic potential at $z = 4$. Express the electric and displacement fields above and below $z = 4$ in terms of V_o and use boundary condition equations on $z = 0$, 4, and 10 m surfaces to relate V_o to pertinent surface charge densities.

3. The gap between a pair of parallel infinite copper plates extends from $z = 0$ to $z = W > 0$ and is initially occupied by vacuum (ϵ_o, μ_o). The plates carry equal and oppositely signed surface charge densities and as a consequence we have a constant electric field $\mathbf{E} = 3\hat{z}\,\frac{\text{V}}{\text{m}}$ in vacuum in the gap region and zero electric field elsewhere.

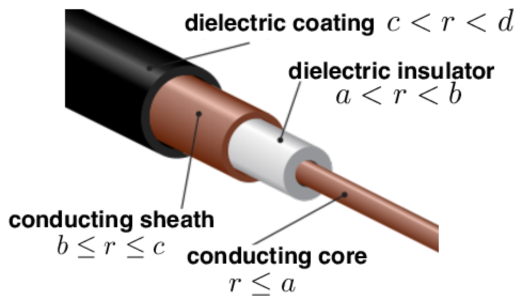
   a) What are the corresponding displacement vector $\mathbf{D}$ and polarization vector $\mathbf{P}$ in the gap region?

   b) What is the surface charge density ρ_s of the copper plate at $z = 0$?

   c) Next, we fill the gap with a non-conducting fluid of permittivity $\epsilon = 81\epsilon_o$ without changing the surface charge densities of the copper plates. What are the new values of $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ in the gap region?

   d) What would be the new equilibrium values of $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ in the gap region if some amount of salt were dissolved in the fluid in the gap (see part c) to raise its conductivity to $\sigma = 4\,\frac{\text{S}}{\text{m}}$ (conductivity of sea water)? State the values of $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ after a steady-state equilibrium is reached and briefly explain your answer.

4. Consider an infinitely long co-axial (cylindrically symmetric) cable, comprised of the following configuration of composite materials in *steady-state equilibrium* (see figure below):
(i) The center core region, defined by $r \le a$ (where $r = \sqrt{x^2+y^2}$ is the radial distance from the origin), is made of a conducting material, having $\epsilon = \epsilon_o$ and $\sigma = 10^6\,\frac{\mathrm{S}}{\mathrm{m}}$, which holds a net charge per unit length of $Q = 2\,\frac{\mathrm{C}}{\mathrm{m}}$.
(ii) Region $a < r < b$ contains a cylindrical shell made of perfect dielectric material having $\epsilon = 10\epsilon_o$
(iii) Region $b \le r \le c$ is occupied by a shell made of conducting material having the same properties as region $r \le a$ and holds a net charge per unit length of $-4\,\frac{\mathrm{C}}{\mathrm{m}}$.
(iv) Region $c < r < d$ contains another perfect dielectric shell having $\epsilon = 2\epsilon_o$.
(v) Region $r \ge d$ is occupied by free space.



a) **D** in all five regions.
b) **E** in all five regions.
c) **P** in all five regions.
d) The surface charge densities, in $\frac{\mathrm{C}}{\mathrm{m}^2}$ units, at each of the four material boundaries at $r = a$, b, c, and d. (**Hint:** Make use of Gauss's law in integral form, $\oint_S \mathbf{D}\cdot dS = \int_V \rho dV$, with $\mathbf{D} = \epsilon\mathbf{E} = \epsilon_o\mathbf{E} + \mathbf{P}$, and a crucial fact about steady-state fields within conducting materials.)
e) Find the capacitance C and conductance G of a 1 m segment of this coax cable.