- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

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**Reading Assignment:** Kudeki: Lectures 6-9
**Recommended Reading:** Staelin: 2.4-2.6, 3.1, 4.5, 5.2

1. Given that $\mathbf{E} = \sin(y)\hat{x} + \cos(x)\hat{y}\ \frac{\text{V}}{\text{m}}$ in free space,
   a) Determine $\nabla \times \mathbf{E}$ and $\nabla\times\nabla\times\mathbf{E}$.
   b) Determine $\rho$ such that Gauss' Law is satisfied.

2. Given an electrostatic potential $V(x, y, z) = x^2 - 2\ V$ in a certain region of free space, determine the corresponding
   a) electrostatic field, $\mathbf{E} = -\nabla V$.
   b) curl, $\nabla \times \mathbf{E}$.
   c) divergence, $\nabla \cdot \mathbf{E}$.
   d) charge density $\rho$ in the region.

3. Given that $V(x, y, z) = x^2 - yz$ and $\mathbf{E} = -\nabla V$, what is $\nabla \times \mathbf{E}$?

4. An important vector identity which is true for any vector field $\mathbf{A}(x, y, z)$ is

   $$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A},$$

   where

   $$\nabla^2\mathbf{A} \equiv (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\mathbf{A}$$

   is the *Laplacian* of $\mathbf{A}$ and $\nabla(\nabla \cdot \mathbf{A})$ is the gradient of the divergence of $\mathbf{A}$.

   Verify the identity for $\mathbf{A} = (x - y)\,\hat{x} + (x + y)\,\hat{y} + 2\,\hat{z}$ by calculating each side of the identity and showing them to be the same.

5. In a given region, $\mathbf{E} = 3x\,\hat{x} + y\,\hat{y} + 2z^2\,\hat{z}\ \frac{\text{V}}{\text{m}}$.
   a) Determine the electrostatic potential $V(1, 2, 3)$ if $V(0, 0, 0) = 0$.
   b) What is the charge density $\rho$ at the points $(0, 0, 0)$ and at $(1, 2, 3)$?

6. Two dielectric media with permittivities $\epsilon_1 = \epsilon_0$ and $\epsilon_2 = 4\epsilon_0$ are separated by a charge free boundary. The electric field in medium 1 has magnitude of 10 $\frac{\text{V}}{\text{m}}$ at an angle 30 degrees from the normal.
   a) Find the magnitude and direction of the electric field in medium 2.
   b) Suppose the electric field in medium 1 is the same, but the electric field in medium 2 is now $10\frac{\text{V}}{\text{m}}$. Find the surface charge on the boundary.
   c) Suppose the boundary is charge-free and medium 2 is now a slab of finite thickness with medium 3 ($\epsilon_3 = \epsilon_0$) on the other side. What is the angle from normal of the electric field in medium 3?

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