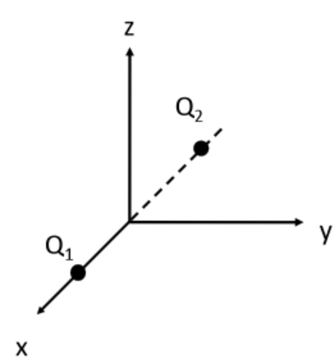
- Homeworks are due Thursdays at 5:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 3-5

1. Gauss' Law for electric field $\mathbf{E}$ states that $\oint_S \mathbf{E} \cdot \mathbf{dS} = \frac{1}{\epsilon_0} \int_V \rho \, dV$ over any closed surface S enclosing a volume V in which electric charge density is specified by $\rho(x, y, z) \; \frac{\mathrm{C}}{\mathrm{m}^3}$.
   a) What is the *electric flux* $\oint_S \mathbf{E} \cdot \mathbf{dS}$ over the surface of a cube of volume $V = L^3$ centered on the origin, if $\rho(x, y, z) = 2 \; \mathrm{C/m}^3$ within V and $L = 2\,\mathrm{mm}$?
   b) Repeat (a) for $\rho(x, y, z) = x^2 + y^2 + z^2 \, \frac{\mathrm{C}}{\mathrm{m}^3}$.
   c) What is the *displacement flux* $\oint_S \mathbf{D} \cdot d\mathbf{S}$ over the surface S in part (b)?
   d) What is the displacement flux in part (b) for any one of the square surfaces of volume V?

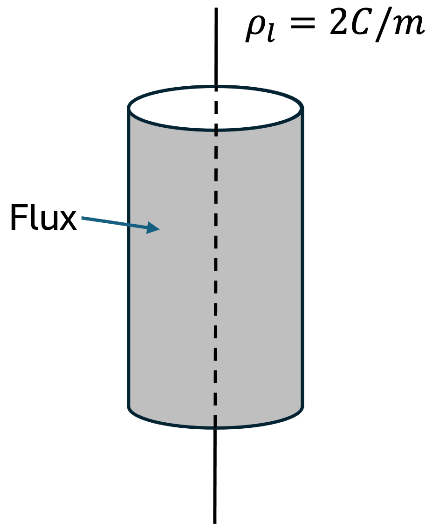
2. Two unknown charges, Q_1 and Q_2 are located at $(x, y, z) = (1, 0, 0)$ and $(-1, 0, 0)$, respectively, as shown below. The displacement flux $\int_{yz-plane} \mathbf{D} \cdot \hat{x} \, dy \, dz$ through the entire $yz-$plane (i.e., the $x = 0$ plane) in the $+\hat{x}$ direction is $-2\,\mathrm{C}$. The displacement flux through the $y = 1$ plane in the $-\hat{y}$ direction is $1\,\mathrm{C}$. Determine Q_1 and Q_2 after writing a pair of algebraic equations relating the above displacement fluxes to Q_1 and Q_2. **Hint:** What is the contribution of Q_1 to the flux $\int_{yz-plane} \mathbf{D} \cdot \hat{x} \, dy \, dz$? See Example 5 in Lecture 3.



3. An infinitely long charge with a line of charge density $\rho_l = 2\ C/m$ is located on z-axis surrounded by empty space.

   a) Consider a cylindrical surface at a radial distance of $r = 1\ m$ from the z-axis and the length of the cylinder in z-axis direction is $L = 2\ m$, calculate the flux integral of displacement vector $\mathbf{D} = \epsilon\mathbf{E}$ over the cylindrical surface (ignore the end caps). Show your steps.

   b) Suppose the line of charge is no longer infinite, but truncated so that only the portion of the line inside the cylinder remains charged (same density ρ_l inside, zero outside). Would the flux integral you calculated in (a) change or not? Explain your reasoning.

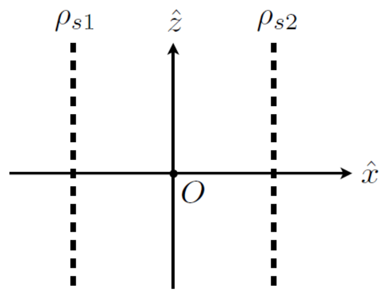


4. Consider two infinite parallel slabs having equal widths W and equal charge densities. Slab 1 is parallel to the xy-plane and extends from $z = -2W$ to $z = -W$ while slab 2 extends from $z = W$ to $z = 2W$. Both slabs have a negative uniform charge density $-\rho_0 \frac{\text{C}}{\text{m}^3}$. The charge density is zero everywhere else.

   a) Determine and sketch the electric field component $\mathbf{E}_z$ in terms of ρ_0 over the region $-3W < z < 3W$ by using shifted and scaled versions of the static field configuration of a single charged slab (See Example 4 in Lecture 3 online). Be sure to label both axes of your plot and mark the field value at each break point.

   b) Verify that your field from part (a) satisfies Gauss' Law in differential form, $\nabla \cdot \mathbf{D} = \rho$ in each slab as well as in the free space regions.

5. An infinite sheet that is uniformly charged with a density $\rho_s \, \frac{\mathrm{C}}{\mathrm{m}^2}$ produces an electrostatic field $\mathbf{E}$ which has a magnitude $\frac{\rho_s}{2\epsilon_0}$ and points away from the sheet on both sides. Using superposition, determine the charge density ρ_{s2} that is required to produce a displacement field $\mathbf{D} \equiv \epsilon_0 \mathbf{E} = 6\hat{x} \, \frac{\mathrm{C}}{\mathrm{m}^2}$ at the origin (labeled O on the figure below), if the charge density ρ_{s1} is given as follows:



a) $\rho_{s1} = 8\frac{\mathrm{C}}{\mathrm{m}^2}$

b) $\rho_{s1} = -\rho_{s2}$

6. **Curl** and **divergence** exercises:

a) On a 25-point graph consisting of x and y coordinates having integer values -2, -1, 0, 1, 2, sketch the vector field $\mathbf{F} = x\hat{x} + y\hat{y}$ and calculate $\nabla \times \mathbf{F}$ (curl of $\mathbf{F}$) and $\nabla \cdot \mathbf{F}$ (divergence of $\mathbf{F}$).

b) Repeat (a) for $\mathbf{F} = y\hat{x} + 2x\hat{y}$.

c) (Select one): If $\nabla \times \mathbf{F} \neq 0$, the field strength varies **along** or **across** the direction of the field.

d) (Select one): If $\nabla \cdot \mathbf{F} \neq 0$, the field strength varies **along** or **across** the direction of the field.