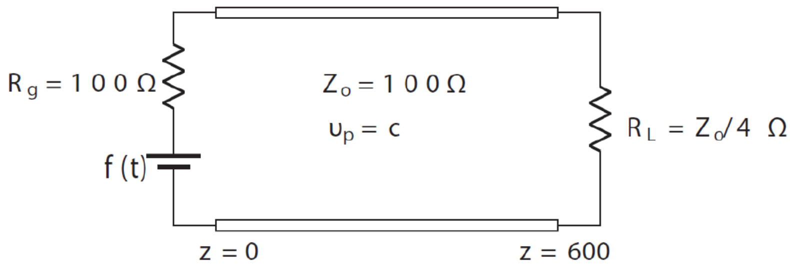
1. The associated circuit is shown in the following figure:



a) In this circuit, the **injection coefficient** is found as

$$\tau_g = \frac{Z_o}{R_g + Z_o} = \frac{1}{2}$$

The load voltage reflection coefficient is given by

$$\Gamma_{L_V} = \frac{R_L - Z_o}{R_L + Z_o} = -\frac{3}{5}$$

whereas the generator voltage reflection coefficient is

$$\Gamma_{g_V} = \frac{R_g - Z_o}{R_g + Z_o} = 0$$

The corresponding current reflection coefficients are

$$\Gamma_{L_I} = -\Gamma_{L_V} = \frac{3}{5}, \text{ and}$$

$$\Gamma_{g_I} = -\Gamma_{g_V} = 0$$

Using these information, we can build the following "bounce diagrams" for the voltage $V(z,t)$ and current $I(z,t)$ on the transmission line.

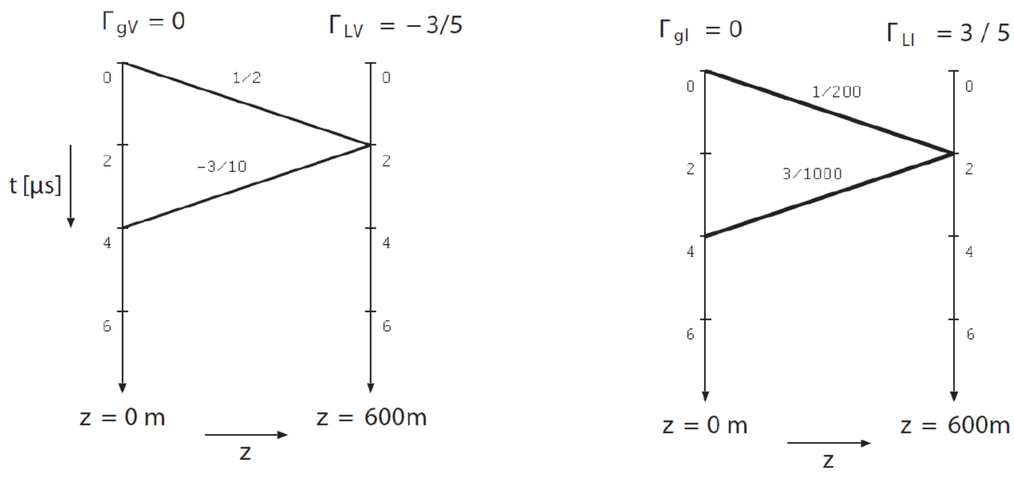


Figure 1: (a) Voltage and (b) current bounce diagrams for voltage source $f(t) = \delta(t)$.

b) The expressions of $V(z,t)$ and $I(z,t)$ are given by

$$V(z,t) = \frac{1}{2}\delta(t-\frac{z}{c}) - \frac{3}{10}\delta(t+\frac{z}{c}-4\mu\text{s})\,\text{V}$$
$$I(z,t) = \frac{1}{200}\delta(t-\frac{z}{c}) + \frac{3}{1000}\delta(t+\frac{z}{c}-4\mu\text{s})\,\text{A}$$

Evaluating these expressions at $z = \frac{l}{4} = 150\,\text{m}$, we have

$$V(\frac{l}{4},t) = \frac{1}{2}\delta(t-0.5\mu\text{s}) - \frac{3}{10}\delta(t-3.5\mu\text{s})\,\text{V}$$
$$I(\frac{l}{4},t) = \frac{1}{200}\delta(t-0.5\mu\text{s}) + \frac{3}{1000}\delta(t-3.5\mu\text{s})\,\text{A}$$

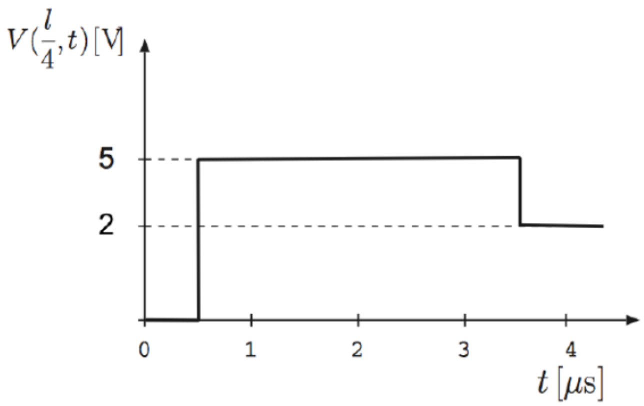
c) Considering the circuit in the problem as a system, its impulse voltage response at $z = \frac{\ell}{4}$ can be written as

$$V(z,t) = h_z(t)\Big|_{z=\frac{\ell}{4}} = \frac{1}{2}\delta(t-0.5\mu\text{s}) - \frac{3}{10}\delta(t-3.5\mu\text{s})\,\text{V}$$

Given that $f(t) = 10u(t)\,\text{V}$, we can compute $V(\frac{l}{4},t)$ as follows

$$V(\frac{l}{4},t) = f(t) * h_z(t) = 5u(t-0.5\mu\text{s}) - 3u(t-3.5\mu\text{s})\,\text{V}$$

The diagram of $V(\frac{\ell}{4},t)$ is shown in the following figure.



d) Given that the voltage source is $f(t) = 10u(t)$, $V(z,t)$ and $I(z,t)$ can also be written as

$$V(z,t) = 5u(t-\frac{z}{c}) - 3u(t+\frac{z}{c}-4\mu\text{s})\,\text{V}$$
$$I(z,t) = \frac{1}{20}u(t-\frac{z}{c}) + \frac{3}{100}u(t+\frac{z}{c}-4\mu\text{s})\,\text{A}$$

By plotting the $V(z,t)$ and $I(z,t)$ as a function of time, we can find the steady state voltage and current as

$$V(z,t) = 5-3 = 2\,\text{V}, \quad \text{and} \quad I(z,t) = \frac{1}{20} + \frac{3}{100} = 0.08\,\text{A}$$

respectively

2. Let us consider the following circuit disgram where Z_o, R_L, and L are unknowns.

   a) Looking at the voltage waveform plot given in the problem, it can be seen that the amplitude of the incident pulse is 60 V. Since the ratio between this amplitude and the source voltage is given by the voltage divider formula, we can write

$$\tau_g = \frac{60}{90} = \frac{2}{3} = \frac{Z_o}{R_g + Z_o} \Longrightarrow Z_o = 100\,\Omega$$

   b) Again looking at the plot in the problem, one can see that the amplitude of the first reflected pulse is -20 V. Therefore, the reflection coefficient is

$$\Gamma_L = -\frac{20}{60} = -\frac{1}{3} = \frac{R_L - Z_o}{R_L + Z_o}$$

   from which we obtain the load resistance as $R_L = \frac{Z_o}{2} = 50\,\Omega$.

   c) The time interval between the incident pulse and the second reflected pulse is $6\,\mu$s, which is equal to the two-way travel time. Then, the time it takes the pulse to travel from one end of the line to the other is $T = 3\,\mu$s.

   d) Since at $z_0 = 300$ m, the incident pulse is delayed by $2\,\mu$s, the propagation speed $v_p = \frac{300}{2\times10^{-6}} = 1.5 \times 10^8$m/s. Thus, we can find that the length of the line is $L = v_p \times T = 150 \times 3 = 450$ m.

   e) The reflected cofficient at the source is $\Gamma_g = -\frac{1}{3}$. Therefore, the next two voltage impluses are $-\frac{20}{9}\delta(t-10)$ V and $\frac{20}{27}\delta(t-14)$ V.

3. Using the figures, we can identify and compute the following parameters of the circuit.

   a) $v_{p1} = \frac{400}{2\times10^{-6}} = 2 \times 10^8$m/s, and $v_{p2} = \frac{300}{3\times10^{-6}} = 1 \times 10^8$m/s.

   b) Since there is no reflection at the loadR_L, the characteristic impedance of the line 2 is $Z_2 = R_L = 60\,\Omega$.

   c) The transmission coefficient from line 1 to line 2 is seen to be $\tau_{12} = \frac{2}{3}$. Hence, the reflection coefficient from line 1 to line 2 is $\Gamma_{12} = \tau_{12} - 1 = -\frac{1}{3}$.

   d) Impedance of line 2 has already been found in part (c) as $Z_2 = 60\,\Omega$. Thus, using $\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -\frac{1}{3}$, we find $Z_1 = 120\,\Omega$.

   e) The reflection coefficient at the source is given by $\Gamma_g = \frac{V^{-,+}}{V^-} = \frac{12}{-20} = -\frac{3}{5}$. Then, utilizing this result in $\Gamma_g = \frac{R_g - Z_1}{R_g + Z_1}$, we find $R_g = \frac{1}{4}Z_1 = 30\,\Omega$.

   f) Using the voltage divider rule, the incident voltage is expressed as $60\text{ V} = V_o \frac{Z_1}{R_g + Z_1}$ where V_o is the source voltage. Therefore, $V_o = 60\frac{R_g + Z_1}{Z_1} = 75$ V.

   g) Reflected voltage $V^{-,+,-} = -4$ V.

   h) Since, as $t \to \infty$, transmission lines become ordinary wires, the steady-state voltage on line 1 is $V_1 = V_o \frac{R_L}{R_g + R_L} = 75 \times \frac{60}{30+60} = 50$ V.

   i) Same as above, the steady-state voltage on line 2 is $V_2 = 50$ V

4. There is a resistor in-between two TLs, so:

a) The voltage in the first line will be a sum of incident $V^+$and reflected voltages V^-, i.e. $V_1 = V^+ + V^-$ whereas after the first line it will be $V_2 = V_R + V^{++}$ where V_R is the voltage across the resistor. Therefore, the KVL equation at the junction is

$$V_1 = V_2 \implies V^+ + V^- = V_R + V^{++}$$

The current flowing in the first line is the sum of the incident and reflected currents, i.e. $I_1 = \frac{V^+}{Z_1} - \frac{V^-}{Z_1}$. However, this current must also be equal to the current flowing through the resistor and also to the current transmitted to the second line. Thus, the KCL equation at the junction is given by

$$\frac{V^+ - V^-}{Z_1} = \frac{V_R}{R} = \frac{V^{++}}{Z_2}$$

b) Combining the previous equations by eliminating V_R we have

$$V^+ - V^- = \left(\frac{Z_1}{Z_2}V^{++}\right)$$

$$V^+ + V^- = \left(\frac{R + Z_2}{Z_2}V^{++}\right)$$

Thus, the reflection coefficient is

$$\Gamma_{12} = \frac{V^-}{V^+} == \frac{R + Z_2 - Z_1}{R + Z_2 + Z_1}$$

Similarly, transmission coefficient is simply found by referring to $\tau_{12} = \frac{V^{++}}{V^+}$. Hence, we have

$$\begin{aligned}\tau_{12} &= (1 + \Gamma_{12})\frac{Z_2}{R + Z_2} \\ &= \frac{2Z_2}{R + Z_2 + Z_1}\end{aligned}$$

where $\frac{Z_2}{R+Z_2}$ is a voltage division factor. Note in this case $\tau_{12} - \Gamma_{12} \neq 1$(because we have an extra resistor.)

(c) Considering $Z_1 = 100\,\Omega$, $Z_2 = 50\,\Omega$, and $R = 25\,\Omega$, we can find that $Z_{eq} = 75\,\Omega$. Then, we calculate

$$\Gamma_{12} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1} = \frac{75 - 100}{75 + 100} = -\frac{1}{7}$$

and

$$\tau_{12} = \frac{2Z_2}{Z_{eq} + Z_1} = \frac{100}{125 + 50} = \frac{4}{7}$$

Note that $1 + \Gamma_{12} \neq \tau_{12}$.

5.

a) Considering that the source voltage is given by $f(t) = u(t)$, it is directly seen from the plot that the injection coefficient $\tau_g = 0.5$. Hence, utilizing $\tau_g = \frac{Z_o}{R_g+Z_o}$, the characteristic impedance of the T.L. is found to be $Z_o = 100\,\Omega$.

b) The plot has a jump at $t = 2\,\text{ns}$, and we can infer that it takes the wave 2 ns to start from the source, then get reflected by the defect, finally go back to the source. Based on this, we have

$$\frac{2d}{v} = 2\,\text{ns}$$

Therefore, $d = 0.2\,\text{m}$.

c) Again, from the plot we know that

$$\begin{aligned} \tau_g \Gamma_d &= 1 - 0.5 \\ 0.5 \times \Gamma_d &= 0.5 \end{aligned}$$

from which we get $\Gamma_d = 1$. (Or, apparently from the plot, a 0.5 V wave goes in, and a 0.5 V wave comes out, which gives a total measurement of 1 V at the source. The reflection coefficient is $0.4/0.4 = 1$).
On the other hand, looking back to the derivation in Problem 4 of this homework, we have

$$\Gamma_d = \frac{(R_d + Z_o) - Z_o}{(R_d + Z_o) + Z_o} = \frac{R_d}{R_d + 2Z_o}$$

where R_d denotes the effective series resistance of the defect. We see R_d should be very/infinitely large $R_d \to \infty$. So the defect is actually an open circuit. (It is helpful to remember an open circuit has reflection coefficient of 1, and a short circuit has reflection coefficient of -1).

d) Since the defect is an open, $\Gamma_d = 1$, $\tau_d = 0$, the wave never reaches the load, and so there will be no voltage response at the load

6.

a) Paragraph 1 – Curiosity: Describe one topic from the course (such as transmission lines, plane waves, or impedance matching) that sparked your curiosity. Explain how you explored this topic beyond lecture material or how it changed your understanding of electromagnetic phenomena.

b) Paragraph 2 – Connections and Value: Discuss how this concept connects to real-world engineering applications (e.g., wireless communication, PCB design, or EMC compliance) and how mastering it helps engineers create value for society through innovation, efficiency, or reliability.