1. For a wave propagating in a vacuum in $x < 0$, we are given an incident electric field phasor

$$\tilde{\mathbf{E}}_i = (j\hat{z} - \hat{y})e^{-j2\pi x}\,\frac{\text{V}}{\text{m}}.$$

The wave encounters a boundary at $x = 0$ with $\mu = \mu_o$ and the reflected phasor is given by

$$\tilde{\mathbf{E}}_r = -\frac{1}{2}(j\hat{z} - \hat{y})e^{j2\pi x}\,\frac{\text{V}}{\text{m}}.$$

a) The incident wave is RHCP because $\hat{y}$ leads $\hat{z}$ and the wave is propagating in the $\hat{x}$ direction. The reflected wave is therefore LHCP.

b) The frequency can be calculated from $f = \frac{v_p \beta}{2\pi}$, where $\beta = 2\pi$ and $v_p = c$ in a vacuum. Thus, $f = 300\,\text{MHz}$.

c) The permittivity of the dielectric can be calculated using the reflection coefficient, Γ.

$$\begin{aligned}
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{1}{2} \\
\eta &= \frac{1}{3}\eta_0 \\
\epsilon_r &= 9 \\
\epsilon &= 9\epsilon_o
\end{aligned}$$

d) The transmitted electric phasor can be derived from the incident electric phasor with updated β and τ. So, $1+\Gamma = \tau = \frac{1}{2}$ and $\beta = \frac{2\pi f}{v_p} = \frac{6\pi\times10^8}{3\times10^8/\sqrt{9}} = 6\pi$ and therefore $\tilde{\mathbf{E}}_t = \frac{1}{2}(j\hat{z} - \hat{y})e^{-j6\pi x}\,\frac{\text{V}}{\text{m}}$.

e) The ratio of time-averaged incident power to time-averaged transmitted power can be found using the conservation of energy: $\frac{\eta_0}{\eta}\tau^2 = 1 - \Gamma^2 = 75\,\%$. The proof is presented below.
The magnetic fields in both the regions are given as,

$$\begin{aligned}
\tilde{\mathbf{H}}_i &= \tfrac{1}{\eta_o}(-j\hat{y} - \hat{z})e^{-j2\pi x} \\
\tilde{\mathbf{H}}_r &= \tfrac{\Gamma}{\eta_o}(-j\hat{y} + \hat{z})e^{j2\pi x} \\
\tilde{\mathbf{H}}_t &= \tfrac{\tau}{\eta}(-j\hat{y} - \hat{z})e^{-j6\pi x}
\end{aligned}$$

The corresponding time averaged poynting vectors ,

$$\begin{aligned}
\langle \mathbf{S}_i \rangle &= \tfrac{1}{2}\text{Re}\left\{\tilde{\mathbf{E}}_i \times \tilde{\mathbf{H}}_i{}^*\right\} = \tfrac{1}{\eta_o}\hat{x} \\
\langle \mathbf{S}_r \rangle &= \tfrac{1}{2}\text{Re}\left\{\tilde{\mathbf{E}}_r \times \tilde{\mathbf{H}}_r{}^*\right\} = \tfrac{\Gamma^2}{\eta_o}\hat{x} \\
\langle \mathbf{S}_t \rangle &= \tfrac{1}{2}\text{Re}\left\{\tilde{\mathbf{E}}_t \times \tilde{\mathbf{H}}_t{}^*\right\} = \tfrac{\tau^2}{\eta}\hat{x}
\end{aligned}$$

It can be shown that the power density calculated for the incident, reflected, and transported waves will satisfy

$$|\langle \mathbf{S}_i \rangle| = |\langle \mathbf{S}_r \rangle| + |\langle \mathbf{S}_t \rangle|\,,$$

which shows that the calculations are in compliance with energy conservation principle. Also, note that

$$\begin{aligned}
\tfrac{1}{\eta_o} &= \tfrac{\Gamma^2}{\eta_o} + \tfrac{\tau^2}{\eta} \\
\tfrac{\eta_0}{\eta}\tau^2 &= 1 - \Gamma^2
\end{aligned}$$

2. The phasor form of the incident wave is $\tilde{\mathbf{E}}_i = -2je^{-j\beta_1 z}\hat{x}\,\frac{\mathrm{V}}{\mathrm{m}}$.

a) The reflection coefficient is given by

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.23 \quad \text{and} \quad \tau = 1 + \Gamma = 0.77.$$

Then, the reflected and transmitted electric fields are expressed as

$$\tilde{\mathbf{E}}_r = -\Gamma 2je^{j\beta_1 z}\hat{x} = 0.46je^{j\beta_1 z}\hat{x}\,\frac{\mathrm{V}}{\mathrm{m}}$$

and

$$\tilde{\mathbf{E}}_t = -\tau 2je^{-j\beta_2 z}\hat{y} = -1.54je^{-j\beta_2 z}\hat{x}\,\frac{\mathrm{V}}{\mathrm{m}}$$

where $\beta_1 = \omega\sqrt{\epsilon_o\mu_o} = \frac{\omega}{c}$ and $\beta_2 = \omega\sqrt{2.56\epsilon_o\mu_o} = 1.6\frac{\omega}{c}$.

b) The associated incident, reflected, and transmitted magnetic fields are

$$\begin{aligned}
\tilde{\mathbf{H}}_i &= -\frac{2}{\eta_1}je^{-j\beta_1 z}\hat{y} = -\frac{1}{60\pi}je^{-j\beta_1 z}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}, \\
\tilde{\mathbf{H}}_r &= \frac{2}{\eta_1}\Gamma je^{j\beta_1 z}\hat{y} = -\frac{0.23}{60\pi}je^{j\beta_1 z}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}},
\end{aligned}$$

and

$$\tilde{\mathbf{H}}_t = -\frac{2}{\eta_2}\tau je^{-j\beta_2 z}\hat{y} = -\frac{1.23}{60\pi}je^{-j\beta_2 z}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}.$$

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c) The time-average Poynting vectors of the incident, reflected, and transmitted waves are given by

$$\begin{aligned}
\langle \mathbf{S}_i \rangle &= \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_i \times \tilde{\mathbf{H}}_i{}^*\right\} = \frac{1}{60\pi}\,\hat{z}\,\frac{\mathrm{W}}{\mathrm{m}^2}, \\
\langle \mathbf{S}_r \rangle &= \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_r \times \tilde{\mathbf{H}}_r{}^*\right\} = -\frac{3}{3380\pi}\,\hat{z}\,\frac{\mathrm{W}}{\mathrm{m}^2}, \\
\langle \mathbf{S}_t \rangle &= \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_t \times \tilde{\mathbf{H}}_t{}^*\right\} = \frac{8}{507\pi}\,\hat{z}\,\frac{\mathrm{W}}{\mathrm{m}^2},
\end{aligned}$$

respectively. It can be seen that they match energy conservation principle.

d) The reflectance is defined as $|\Gamma^2| = \frac{|\langle \mathbf{S}_r \rangle|}{|\langle \mathbf{S}_i \rangle|}$. In order to express it in dB, we write

$$\begin{aligned}
10\log_{10}|\Gamma^2| &= 10\log_{10} 0.23^2 = 10\log_{10}(0.053) \\
&= -12.77\,\mathrm{dB}.
\end{aligned}$$

Therefore, the reflectance of an interface indicates the ratio of the reflected and the incident powers. For this case, the reflected signal level is 12.77 dB below the incident signal level, which is equivalent to 5.3% in power and 23% in amplitude compared to the incident signal.

3. A plane wave field

$$\mathbf{H}(y,t) = \hat{x}5cos(\omega t + \beta y)\,\frac{\mathrm{A}}{\mathrm{m}}$$

is propagating in a dielectric in the region $y > 0$. Here, $\epsilon = 4\epsilon_0$ and thus $\eta = \frac{1}{2}\eta_0$. At $x = 0$, there is a boundary to a perfect conductor where $\eta = 0$. Then the reflection and transmission coefficients can be found to be

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \frac{1}{2}\eta_0}{0 + \frac{1}{2}\eta_0} = -1 \quad \text{and} \quad \tau = 1 + \Gamma = 0.$$

a) It can be shown that the incident, reflected, and transmitted electric fields are

$$\tilde{\mathbf{E}}_i = -\hat{z}\frac{5\eta_0}{2}e^{j\beta y}\,\frac{\mathrm{V}}{\mathrm{m}},$$

$$\tilde{\mathbf{E}}_r = -\hat{z}\Gamma\frac{5\eta_0}{2}e^{-j\beta y} = \hat{z}\frac{5\eta_0}{2}e^{-j\beta y}\,\frac{\mathrm{V}}{\mathrm{m}},$$

and

$$\tilde{\mathbf{E}}_t = \hat{z}\tau\frac{5\eta_0}{2}e^{j\beta_{PEC}x} = 0\,\frac{\mathrm{V}}{\mathrm{m}}.$$

b) The associated incident, reflected, and transmitted magnetic fields are

$$\tilde{\mathbf{H}}_i = \hat{x}5e^{j\beta y}\,\frac{\mathrm{A}}{\mathrm{m}},$$

$$\tilde{\mathbf{H}}_r = \hat{x}5e^{-j\beta y}\,\frac{\mathrm{A}}{\mathrm{m}},$$

and

$$\tilde{\mathbf{H}}_t = 0\,\frac{\mathrm{A}}{\mathrm{m}}.$$

c) The vector current density on the surface of the PEC is found using boundary conditions:

$$\begin{aligned}
\mathbf{J}(t) &= \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) \\
&= \hat{n} \times (0 - (\mathbf{H_i}|_{x=0} + \mathbf{H_r}|_{x=0})) \\
&= -\hat{y} \times -(\hat{x}5cos(\omega t) + \hat{x}5cos(\omega t)) \\
&= -10cos(\omega t)\hat{z}\,\frac{\mathrm{A}}{\mathrm{m}}.
\end{aligned}$$

4.

a) The geometrical factor of the RG-59 coax cable is given by

$$\mathrm{GF} = \frac{2\pi}{\ln\left(\frac{0.056}{0.016}\right)} = 5.015,$$

from which we obtain

$$\mathcal{L} = \frac{\mu_o}{\mathrm{GF}} = \frac{4\pi \times 10^{-7}}{2\pi/\ln(3.5)} = 250.6\,\frac{\mathrm{nH}}{\mathrm{m}},$$

$$\mathcal{C} = \epsilon_o \mathrm{GF} \approx \frac{10^{-9}}{36\pi}\frac{2\pi}{\ln(3.5)} = 44.3\,\frac{\mathrm{pF}}{\mathrm{m}},$$

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu_o}{\epsilon_o}} \approx \frac{\ln(3.5)}{2\pi}120\pi = 75.17\,\Omega,$$

$$v = \frac{1}{\sqrt{\epsilon_o\mu_o}} = c \approx 3 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}}.$$

b) The geometrical factor of the RG-58 coax cable will not change since RG-58 has the same inner and outer conductor diameters. On the other hand, the inductance, capacitance, characteristic impedance, and the propagation velocity will be

$$\begin{aligned}
\mathcal{L} &= \frac{\mu_o}{\mathrm{GF}} = \frac{4\pi \times 10^{-7}}{2\pi/\ln(3.5)} = 2.506 \times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}} = 250.6\,\frac{\mathrm{nH}}{\mathrm{m}}, \\
\mathcal{C} &= 2.25\epsilon_o \mathrm{GF} \approx \frac{2.25 \times 10^{-9}}{36\pi}\frac{2\pi}{\ln(3.5)} = 99.8\,\frac{\mathrm{pF}}{\mathrm{m}}, \\
Z_o &= \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu_o}{2.25\epsilon_o}} \approx \frac{\ln(3.5)}{2\pi}\frac{120\pi}{\sqrt{2.25}} = 50.11\,\Omega, \\
v &= \frac{1}{\sqrt{2.25\epsilon_o\mu_o}} = \frac{2}{3}c \approx 2 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}}.
\end{aligned}$$

due to the change in the permittivity of the dielectric filling.

5. For twin-lead transmission lines, the geometrical factor is given by

$$\mathrm{GF} = \frac{\pi}{\cosh^{-1}\left(\frac{D}{2a}\right)}.$$

Since

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu}{\epsilon}} = \frac{\cosh^{-1}\left(\frac{D}{2a}\right)}{\pi}\sqrt{\frac{\mu}{\epsilon}},$$

we can can find that

$$D = 2a \cosh\left(Z_o \pi \sqrt{\frac{\epsilon}{\mu}}\right).$$

Assuming $\epsilon = \epsilon_o$, $\mu = \mu_o$, and $2a = 1\,\mathrm{mm}$, let us calculate D as follows:

a) For $Z_o = 150\,\Omega$,

$$D = 1 \times 10^{-3} \cosh\left(\frac{150\pi}{120\pi}\right) \approx 1.89 \times 10^{-3}\,\mathrm{m} = 1.89\,\mathrm{mm}.$$

b) For $Z_o = 300\,\Omega$,

$$D = 1 \times 10^{-3} \cosh\left(\frac{300\pi}{120\pi}\right) \approx 6.13 \times 10^{-3}\,\mathrm{m} = 6.13\,\mathrm{mm}.$$

c) For $Z_o = 450\,\Omega$,

$$D = 1 \times 10^{-3} \cosh\left(\frac{450\pi}{120\pi}\right) = 21.27 \times 10^{-3}\,\mathrm{m} = 21.27\,\mathrm{mm}.$$

6. The voltage and current waves $V(z,t)$ and $I(z,t)$ that propagate on a transmission line satisfy the following set of partial differential equations (PDE's)

$$
\begin{aligned}
-\frac{\partial V}{\partial z} &= \mathcal{L}\frac{\partial I}{\partial t}, && (1)\\
-\frac{\partial I}{\partial z} &= \mathcal{C}\frac{\partial V}{\partial t}. && (2)
\end{aligned}
$$

Given that $V(z,t) = 4\cos(\omega t + \beta z)$, we get

$$\frac{\partial I}{\partial t} = -\frac{1}{\mathcal{L}}\frac{\partial V}{\partial z} = 4\frac{\beta}{\mathcal{L}}\sin(\omega t + \beta z),$$

by utilizing (1). Then, after integrating it over time, we get

$$I = -4\frac{\beta}{\omega\mathcal{L}}\cos(\omega t + \beta z).$$

Inserting this result into (2), we get

$$\frac{\partial V}{\partial t} = -\frac{1}{\mathcal{C}}\frac{\partial I}{\partial z} = -4\frac{\beta^2}{\omega\mathcal{L}\mathcal{C}}\sin(\omega t + \beta z),$$

and integrating over time, we finally obtain

$$V = 4\frac{\beta^2}{\omega^2\mathcal{L}\mathcal{C}}\cos(\omega t + \beta z).$$

This result implies that

$$\frac{\beta^2}{\omega^2\mathcal{L}\mathcal{C}} = 1, \qquad \text{then} \qquad \beta = \omega\sqrt{\mathcal{L}\mathcal{C}}.$$

In addition, the expression for the current becomes

$$I = -4\sqrt{\frac{\mathcal{C}}{\mathcal{L}}}\cos(\omega t + \beta z).$$