ECE 329 **Homework 1** **Due: Friday, August 29, 2025, 4:59:00PM**

- Homeworks are due Fridays at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

Reading Assignment: Kudeki: Lectures 1-3, "Review of Vector Calculus" under "Supplementals" link on course web site

1. Consider the vectors in 3-D Cartesian coordinates:

$$\mathbf{A} = \hat{x} + 3\hat{y} - 2\hat{z},$$

$$\mathbf{B} = \hat{x} + \hat{y} - \hat{z},$$

$$\mathbf{C} = -\hat{x} + 3\hat{y} + 2\hat{z},$$

where $\hat{x} \equiv (1,0,0)$, $\hat{y} \equiv (0,1,0)$, and $\hat{z} \equiv (0,0,1)$ constitute an orthogonal set of unit vectors directed along the principal axes of a *right-handed* Cartesian coordinate system. Vectors can also be represented in component form — e.g., $\mathbf{A} = (1,3,-2)$, which is equivalent to $\hat{x} + 3\hat{y} - 2\hat{z}$.
Determine:

   a) The vector $\mathbf{A} - \mathbf{B} + 3\mathbf{C}$.
   b) The vector *magnitude* $|\mathbf{A} - \mathbf{B} + 3\mathbf{C}|$.
   c) The unit vector $\hat{u}$ along vector $\mathbf{A} + 2\mathbf{B} - \mathbf{C}$.
   d) The *dot products* $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{B} \cdot \mathbf{A}$.
   e) The angle θ between $\mathbf{A}$ and $\mathbf{B}$.
   f) The *cross products* $\mathbf{B} \times \mathbf{C}$ and $\mathbf{C} \times \mathbf{B}$.
   g) Two of the vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are orthogonal. Specify which two and explain why.

2. Charges $Q_1 = -1\,\mathrm{C}$ and $Q_2 = +1\,\mathrm{C}$ are located at points P_1 and P_2 having the position vectors $\mathbf{r}_1 = -\hat{x} = (-1,0,0)$ m and $\mathbf{r}_2 = \hat{x} = (1,0,0)$ m, respectively.

   a) Determine the electric field vector $\mathbf{E}$ at points P_3, P_4, and P_5 having the position vectors $\mathbf{r}_3 = (0,0,0)$ m, $\mathbf{r}_4 = 2\hat{x} = (2,0,0)$ m, and $\mathbf{r}_5 = -\hat{z} = (0,0,-1)$ m, respectively.
   b) Consider a charge $Q = 2\,\mathrm{C}$ placed at $(0,0,0)$. What is the force $\mathbf{F}$ experienced by the charge?

3. A particle with charge $q = 1$ C passing through the origin $\mathbf{r} = (x,y,z) = 0$ of the lab frame is observed to accelerate with forces

$$\mathbf{F}_1 = 2\hat{x} - \hat{y}, \quad \mathbf{F}_2 = 2\hat{x} - 3\hat{y}, \quad \mathbf{F}_3 = 3\hat{x} - 2\hat{y} - \hat{z} \quad \mathrm{N}$$

when the velocity of the particle is

$$\mathbf{v}_1 = 0, \quad \mathbf{v}_2 = -2\hat{z}, \quad \mathbf{v}_3 = \hat{x} + \hat{y} \quad \frac{\mathrm{m}}{\mathrm{s}},$$

in turns. Use the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ to determine the fields $\mathbf{E}$ and $\mathbf{B}$ at the origin.
Hint: Assume the $\mathbf{B}$ and $\mathbf{E}$ fields are the same for all three measurements, then consider a $\mathbf{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ and solve the three vector equations obtained from $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ for the unknowns B_x, B_y, B_z, and $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$.

4. Consider a scalar field

$$V = x^2 + y^2 + z^2$$

   a) Find ∇V.

b) What is ∇V at (1,2,3)?

c) What does ∇V represent geometrically?

5. Consider a vector field

$$\mathbf{E} = \hat{x}xy + \hat{y}yz + \hat{z}zx$$

a) Find $\nabla \cdot \mathbf{E}$.

b) What is $\nabla \cdot \mathbf{E}$ at (1,1,1)?

c) Find $\nabla \times \mathbf{E}$.

d) Find $\nabla \cdot (\nabla \times \mathbf{E})$.

e) Find $\int_{(0,0,0)}^{(1,1,1)} \mathbf{E} \cdot d\mathbf{l}$ along the path $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$.