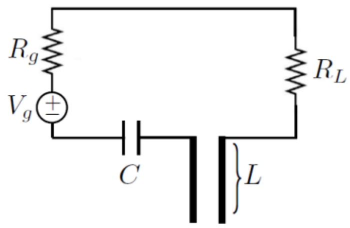
Reading Assignment: Kudeki: Lectures 33-35

1. Consider the circuit shown below, where a resistor, $R_L = 50\ \Omega$, and a capacitor C, are connected in series with an open-circuited transmission line stub with characteristic impedance $Z_0 = R_L$, $v = c$, and length $L = 75$ cm (see diagram).



a) As derived in the previous question, the current at the TL ends must be zero

$$V^+ = V^-.$$

Therefore, the voltage and the current expressions are given by

$$\tilde{V}(z) = 2V^+ \cos(\beta z)$$

and

$$\tilde{I}(z) = -\frac{V^+}{Z_o} 2j \sin(\beta z).$$

Therefore, the input impedance of open circuit TL stub is given by

$$Z_{\text{in}} = \frac{\tilde{V}(-L)}{\tilde{I}(-L)} = -jZ_o \cot(\beta L).$$

Keep in mind the convention used here is to place the load at $z = 0$ and source at $z = -L$. Alternatively, we can say the load is placed at $d = 0$ and source is $d = +L$, in which case, current and impedance will be positive, but the resulting Capacitance in part (b) will be the same. Remember to stay consistent!

b) By steady state analysis, we can regard all circuit elements as having general impedance. Thus, the impedance of the capacitor will be

$$Z_C = \frac{1}{j\omega C}.$$

Using the voltage division rule, we write

$$V_L = V_g \frac{R_L}{R_L + R_g + Z_{\text{in}} + Z_C}.$$

Given that $R_g = R_L$ and $V_L = \frac{1}{2}V_g$, we find $Z_{\text{in}} + Z_C = 0$. Therefore, we can write

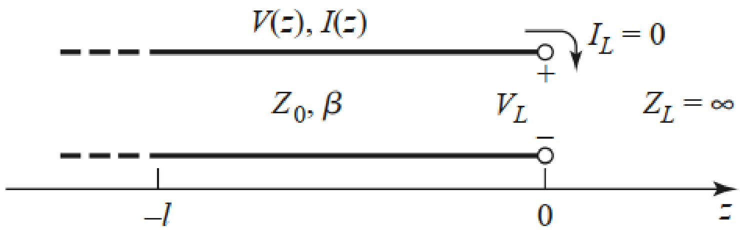
$$-jZ_o \cot(\beta L) = \frac{1}{j\omega C}$$

from which we obtain

$$C = -\frac{1}{\omega Z_o \cot(\beta L)} = -\frac{1}{\omega Z_o \cot(\frac{\omega}{v} L)} = -63.66\,\text{pF}.$$

Note: Capacitance should be positive. The length in the problem is not practical.

2. Consider the open-circuited transmission line shown in the figure below.



a) For the open-circuited line, the current at the TL ends must be zero

$$V^+ = V^-.$$

Therefore, the voltage and the current expressions are given by

$$\tilde{V}(z) = 2V^+ \cos(\beta z)$$

and

$$\tilde{I}(z) = -\frac{V^+}{Z_o} 2j \sin(\beta z).$$

Therefore, the input impedance of open circuit TL stub is given by

$$Z_{\text{in}} = \frac{\tilde{V}(-l)}{\tilde{I}(-l)} = -jZ_o \cot(\beta l).$$

Keep in mind the convention used here is to place the load at $z = 0$ and source at $z = -l$. Alternatively, we can say the load is placed at $d = 0$ and source is $d = +l$, in which case, current will be positive, but the impedance will be the same.

b) The transmission line length can be obtained using the following:

$$Z_{in} = \frac{1}{j\omega C} = -j12.73\Omega = -jZ_o \cot(\beta l)$$

Hence,

$$\beta l = 1.444\,\text{rad}$$

Using $\lambda_o = c/f = 0.12\,\text{m}$, and $\beta = 2\pi\sqrt{\varepsilon_r}/\lambda_o = 67.257\,\text{rad}$:

$$l = 2.147\,\text{cm}$$

c) The transmission line length can be obtained using the following:

$$Z_{in} = j\omega L = +j78.5\Omega = -jZ_o \cot(\beta l)$$

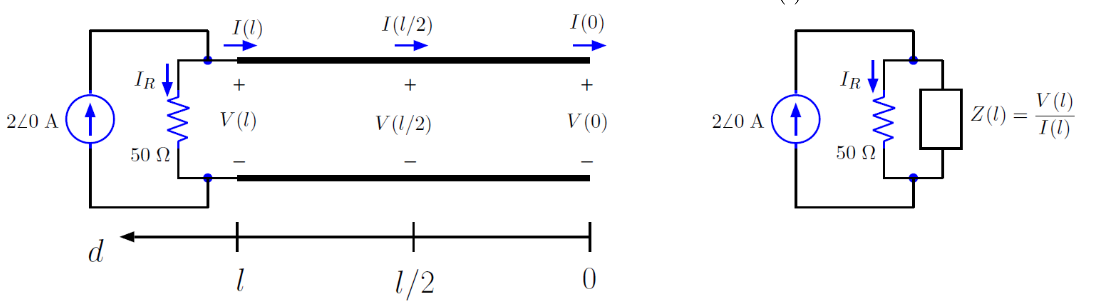
Hence,

$$\beta l = 2.236\,\text{rad}$$

Using the same value for propagation constant β:

$$l = 3.324\,\text{cm}$$

3. Consider a transmission line segment of propagation velocity $v = \frac{1}{3}c = 1 \times 10^8$ m/s, characteristic impedance $Z_o = 50\,\Omega$, and length l. As shown in figure (a) below, the segment is connected *in parallel* with a 50 Ω **resistor** and an ideal **current source** $i(t) = \text{Re}\{Ie^{j\omega t}\}$, where $I = 2\angle 0$ A is the source current phasor and $\omega = \pi \times 10^8$ rad/s; figure (b) depicts an *equivalent circuit* in terms of input impedance of the transmission line at $d = l$, namely $Z(l) \equiv \frac{V(l)}{I(l)}$.



(a) Transmission line circuit (b) Equivalent circui

In answering the following questions assume that the circuit above is in sinusoidal steady-state:

a) Since the end of the line (i.e. at $d = 0$) is open circuited, $I(0) = 0$.

b) If $I_R = 2\,\text{A}$ then the current in the transmission line at $d = l$ is zero (i.e., $I(l) = 0$). Since successive current nulls are $\lambda/2$ apart, the smallest non-zero transmission line length is $l = \lambda/2 = 1\,\text{m}$.

c) If $l = \frac{\lambda}{2}$ then the voltage at $d = l/2$ is zero (i.e, $V(\frac{l}{2}) = 0$), because half-way between successive current nulls there is a voltage null.

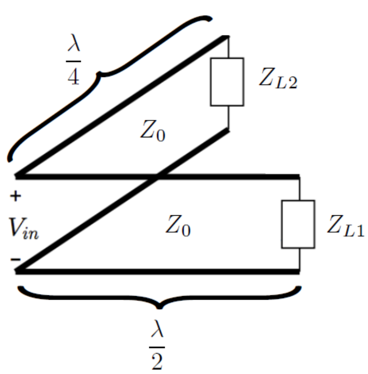
d) If $l = \frac{\lambda}{2}$ then the current at $d = l/2$ can not be zero (i.e., $I(\frac{l}{2}) \neq 0$), because at this point the current is a maximum.

e) If $I_R = 0$ then the voltage at $d = l$ is also zero (i.e., $V(l) = 0$). Since voltage and current nulls are $\lambda/4$ apart, the smallest non-zero transmission line length is $l = \frac{\lambda}{4} = 0.5\,\text{m}$.

f) If $I(\frac{l}{2}) = 0$ and $I(0) = 0$ then $\frac{l}{2} = \frac{\lambda}{2}$ because the separation between two successive current nulls is $\lambda/2$ (non-trivial case). Therefore, $l = \lambda = 2\,\text{m}$.

g) In part (h), $I(\frac{l}{2}) = 0$ and $l = \lambda$ which implies that $I(l) = 0$. Since $I_R = 2\angle 0°$ A then $V(l) = I_R \times 50\,\Omega = 100\angle 0°$ V.

4. Two transmission line segments each having characteristic impedance $Z_0 = 100\ \Omega$ are connected in parallel and share input voltage $V_{in} = j10$ V. One transmission line has electrical length $l = \frac{\lambda}{2}$ and is terminated by a resistive load $Z_{L1} = 200\ \Omega$, while the other has electrical length $l = \frac{\lambda}{4}$ is terminated by $Z_{L2} = 50\ \Omega$.



a) Voltage input at the load end of the half-wave transformer

$$V_{L_1} = -V_{in} = -j10\,V.$$

b) Voltage input at the load end of the quarter-wave transformer

$$V_{L_2} = -j\frac{V_{in}Z_{L_2}}{Z_o} = 5\,V.$$

c) Current input at the load end of the half-wave transfomer

$$I_{L_1} = \frac{V_{L_1}}{Z_{L_1}} = \frac{-j}{20}\,A.$$

d) Current input at the load end of the quarter-wave transformer

$$I_{L_2} = -j\frac{V_{in}}{Z_0} = \frac{V_{L_2}}{Z_{L_2}} = \frac{1}{10}\,A.$$

e) Input impedance of the combined network is given as

$$Z_{in} = Z_{in_1}||Z_{in_2} = Z_{L_1}||\frac{Z_o^2}{Z_{L_2}} = 100\,\Omega.$$

f) Total time-averaged power absorbed is

$$\begin{aligned} P_{avg} &= \tfrac{1}{2}Re\{V_{L_1}I_{L_1}^*\} + \tfrac{1}{2}Re\{V_{L_2}I_{L_2}^*\} \\ &= \tfrac{1}{2}Re\{-j10 \times \tfrac{j}{20}\} + \tfrac{1}{2}Re\{(5) \times (\tfrac{1}{10})\} = 0.5\,W. \end{aligned}$$

5. A quarter-wavelength long transmission line section having characteristic impedance $Z_o = 100\,\Omega$ is terminated by an unknown impedance Z_L at one end. The input current phasor at the other end is $I_{in} = 0.5\angle 0^\circ$ A. Let V_{in} and V_L denote input and load voltage phasors, respectively, with directions defined in a compatible way with one another and with I_{in}.

   a) Referring to page 3 of Lecture 33, the input current is given by

   $$I_{in} \equiv I(\frac{\lambda}{4}) = \frac{jV^+ + jV^-}{Z_o} = j\frac{V(0)}{Z_o} = j\frac{V_L}{Z_o}$$

   Therefore

   $$V_L = -jI_{in}Z_o = -j50\,V.$$

   b) If the load end is shorted then $V_L = 0$. This implies that the current input must also be zero ($I_{in} = j\frac{V_L}{Z_o} = 0$), and therefore, $I_{in} = 0.5\angle 0^\circ$ A is not a possibility in this situation.

   c) Since the voltage measured at any distance along the TL is given by

   $$V(d) = V^+(e^{j\beta d} + \Gamma_L e^{-j\beta d}),$$

   where $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$, the voltages at one end is

   $$V_L = V(0) = V^+(1 + \Gamma_L)$$

whereas the input voltage of the quarter-wave section at the other end is

$$V_{in} = V(\frac{\lambda}{4}) = jV^+(1 - \Gamma_L).$$

Therefore, we can write

$$\frac{V_L}{V_{in}} = -j\frac{1 + \Gamma_L}{1 - \Gamma_L} = -j\frac{Z_L}{Z_o},$$

from which we can obtain

$$V_{in} = j\frac{V_L Z_o}{Z_L} = 100\,V.$$

6. In the transmission-line circuit shown below all lines are lossless, $2Z_{01} = Z_{02} = 100\,\Omega$ and $R_{L1} = R_{L2} = 50\,\Omega$. Calculate the following:

   a) Since the $\frac{\lambda}{2}$-transformer inverts the sign of its voltage and current inputs at the load end, and two $\frac{\lambda}{2}$ transformers will be a double negative. We can write

   $$V_{L1} = -V_g = -100\,V$$

   which implies that

   $$I_{L1} = \frac{V_{L1}}{R_{L1}} = -2\,A$$

   We know that a $\frac{\lambda}{4}$-transformer has a load current $I_L = -j\frac{V_{in}}{Z_o}$. It follows that

   $$\begin{aligned} V_{L2} &= I_{L2}R_{L2} \\ &= -j\frac{V_{in}}{Z_{o2}}R_{L2} = -j50\,V \end{aligned}$$

   which yields

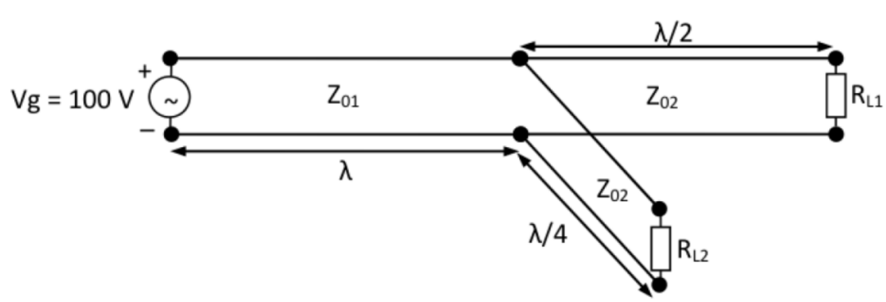
   $$I_{L2} = \frac{V_{L2}}{R_{L2}} = -j\frac{V_{in}}{Z_{o2}} = -j\,A$$

   b) The power dissipated in resistor R_{L1} is

   $$P_{L1} = \frac{1}{2}\text{Re}\left\{V_{L1}I_{L1}^*\right\} = \frac{1}{2}Re\left\{\frac{|V_{L1}|^2}{R_{L1}^*}\right\} = 100\,W$$

   Then for the second line which is a $\frac{\lambda}{4}$-transformer, the time average dissipated power in resistor R_{L2} is given by

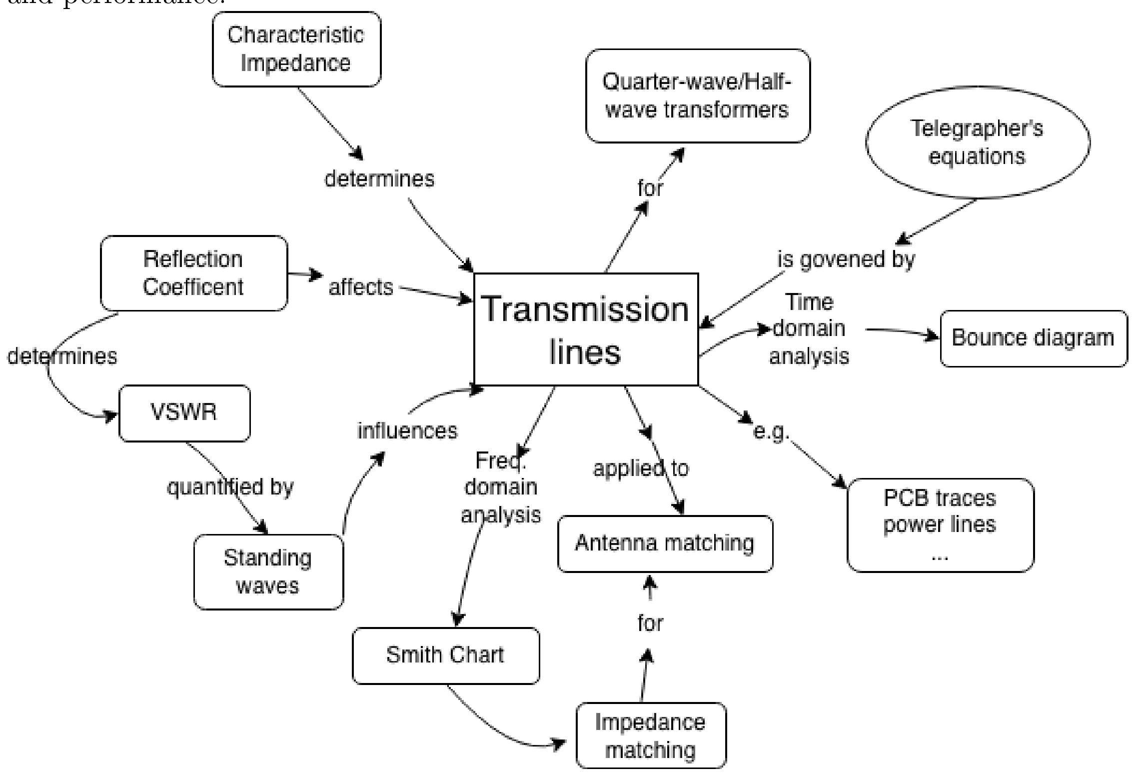
   $$P_{L2} = \frac{1}{2}\text{Re}\left\{V_{L2}I_{L2}^*\right\} = 25\,W$$



7. This homework activity helps you visualize how key ECE 329 topics are connected and how they apply to real engineering problems.

   a) Have you encountered concept map before? Where or in which class?

   Review the sample concept map below showing "Transmission Lines" at the center, connected to ideas such as characteristic impedance, reflection coefficient, standing waves, impedance matching. Note how the map:
   • Uses labeled arrows (e.g., determines, affects, used for) to show relationships
   • Links core theory to real-world applications (e.g., PCB traces, RF mixers, antenna matching)
   • Highlights how understanding these links helps engineers improve system efficiency, reliability, and performance.



   b) Build your own concept map centered on one major ECE 329 topic, such as:
   • Plane waves and reflection at boundaries
   • Transmission-line behavior and impedance matching
   • Power flow and standing-wave ratio (SWR)
   • Polarization of electromagnetic waves
   • Smith Chart as a visualization and design tool
   • Maxwell's equations and field relationships.

   Your map should:
   • Include 8–12 nodes (key ideas, equations, or examples)
   • Use connecting phrases to describe relationships (causes, depends on, used in, etc.)
   • Include at least two links to real-world systems (e.g., wireless links, PCB interconnects, shielding)
   • Show how mastering these concepts creates value for your team/project/company/society (pick

one or come up with your own)—through innovation, performance, or reliability You may hand-draw or use a digital tool (Lucidchart, draw.io, PowerPoint, etc.).