- Homeworks are due Friday at 4:59:00 p.m. Late homework will not be accepted.
- Homeworks are to be turned in to Gradescope.
- Any deviation from the following steps will result in a 5 point penalty:
  - Write your name, netID, and section on each page of your submission.
  - Start each new problem on a new page.
  - Scan the homework as a PDF (rather than taking pictures). Use a free scanning app if you do not have access to a photocopier.
  - Upload the PDF to Gradescope and tag each problem's location in the PDF.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: a zero for the assignment on the first offense, and an F in the course on the second offense.

Reading Assignment: Kudeki: Lectures 27-29

1. A monochromatic plane wave propagating in a vacuum in the region $x < 0$ has an electric field phasor given by

$$\tilde{\mathbf{E}}_{\mathbf{i}} = (j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/m}$$

The wave encounters a planar boundary at $x = 0$ which separates the vacuum from a perfect dielectric material in the region $x > 0$ which has magnetic permeability $\mu = \mu_0$ and electric permittivity $\epsilon > \epsilon_0$. The electric field phasor of the reflected wave in the $x < 0$ region is given by

$$\tilde{\mathbf{E}}_{\mathbf{r}} = -\frac{1}{2}(j\hat{z} - \hat{y})e^{j2\pi x} \text{ V/m}$$

   a) What are the polarizations of the incident and reflected waves?
   b) What is the linear frequency f of the wave?
   c) What is the permittivity ϵ of the dielectric material?
   d) Write the phasor expression for the electric field waveform that is transmitted into the dielectric medium.
   e) What percentage of the time-averaged incident power per unit area is transmitted into the dielectric medium?
   f) What are the time-average power per unit area transported by the incident, reflected, and transmitted waves in W/m^2 units. Are the calculations compatible with energy conservation principle? Explain.

2. A plane wave field

$$\mathbf{E} = 2\sin(\omega t - \beta z)\hat{x}\,\frac{\text{V}}{\text{m}}$$

is propagating in vacuum in $+z$ direction and is incident on $z = 0$ plane which happens to be the boundary of a perfect dielectric having permittivity $2.56\epsilon_o$ and permeability μ_o in the region $z > 0$. Calculate:

   a) The reflected and transmitted electric fields in terms of reflection and transmission coefficients $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ and $\tau = 1 + \Gamma$ for the described interface. Express the fields in phasor form.
   b) The incident, reflected, and transmitted **H** fields in A/m units. Express the fields in phasor form.
   c) The time-average power per unit area transported by the incident, reflected, and transmitted waves in W/m^2 units. Are the calculations compatible with energy conservation principle? Explain.
   d) *Reflectance* $|\Gamma|^2$ expressed in dB units. What would be the physical significance of reflectance? **Hint:** think of it as a fraction.

3. A monochromatic plane wave described by

$$\mathbf{H}(y, t) = \hat{x}5\cos(\omega t + \beta y) \text{ A/m}$$

is propagating in the region $y > 0$, which is a non-magnetic perfect dielectric having permittivity $4\epsilon_0$. The wave is incident on the $y = 0$ plane which happens to be the boundary with a perfect conductor $(\sigma = \infty)$ in the region $y < 0$.

a) Write the phasor expressions for the incident, reflected, and transmitted electric fields $\tilde{\mathbf{E}}_{\mathbf{i}}$, $\tilde{\mathbf{E}}_{\mathbf{r}}$, and $\tilde{\mathbf{E}}_{\mathbf{t}}$.

b) Write the phasor expressions for the incident, reflected, and transmitted magnetic fields $\tilde{\mathbf{H}}_{\mathbf{i}}$, $\tilde{\mathbf{H}}_{\mathbf{r}}$, and $\tilde{\mathbf{H}}_{\mathbf{t}}$.

c) What is the vector current density $\mathbf{J}_{\mathbf{s}}(t)$ generated on the surface of the perfect conductor, i.e., at $y = 0$?

4. RG-58 is the coax cable that you have been using most frequently in your labs. It has the same geometrical dimensions as the RG-59 cable, but instead of having a dielectric filling with $\epsilon = \epsilon_o$, $\mu = \mu_o$ (like RG-59), it has $\epsilon = 2.25\epsilon_o$, $\mu = \mu_o$. The inner and outer conductor diameters for both types of cables are $2a = 0.032$ inches and $2b = 0.112$ inches, respectively. Given that

$$\mathcal{C} = \epsilon \mathrm{GF}, \quad \mathcal{L} = \frac{\mu}{\mathrm{GF}}, \quad Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}, \quad v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}},$$

and geometrical factor

$$\mathrm{GF} = \frac{2\pi}{\ln \frac{b}{a}}$$

for a coax,

a) Calculate, $\mathcal{L}$, $\mathcal{C}$, Z_o, v for the RG-59 coax cable.

b) Repeat (a) for RG-58.

5. $300\,\Omega$ twin-lead transmission lines (TL) are commonly used to connect TV sets and FM radios to their receiving antennas. For the twin-lead, the geometrical factor is

$$\mathrm{GF} = \frac{\pi}{\cosh^{-1} \frac{D}{2a}},$$

where $2a$ is the diameter of each wire (cylindrical conductor) of the twin lead, and D is the distance between the centers of the wires. Assuming $\epsilon = \epsilon_o$, $\mu = \mu_o$, and $2a = 1$ mm, calculate D for twin lead TL's having (a) $Z_o = 150\,\Omega$, (b) $Z_o = 300\,\Omega$.

6. Telegrapher's equations

$$\begin{aligned} -\frac{\partial V}{\partial z} &= \mathcal{L}\frac{\partial I}{\partial t} \\ -\frac{\partial I}{\partial z} &= \mathcal{C}\frac{\partial V}{\partial t} \end{aligned}$$

govern the voltage and current waves $V(z,t)$ and $I(z,t)$ that propagate on transmission line systems.

If $V(z,t) = 4\cos(\omega t + \beta z)$ on a TL, determine $I(z,t)$ and β (a positive number) by using the telegrapher's equations twice.

Hint: First use one of the telegrapher's equations to determine $I(z,t)$. Then use the other telegrapher's equation to determine $V(z,t)$ from $I(z,t)$ found in the first step. By requiring $V(z,t)$ found in step 2 to equal the original $V(z,t)$ you should be able to identify β.