University of Illinois Summer 2017

ECE 313: Exam I

Thursday, July 6, 2017
5.15 - 6.30 p.m.
3015 ECEB

Name: (in BLOCK CAPITALS) ________________________________

NetID: ________________________________

Signature: ________________________________

Instructions

This exam is printed **double-sided**, so make sure to look at all problems and both sides of every sheet.

This exam is closed book and closed notes except that one 8.5”×11” sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the blank page at the end of the exam.

Grading

1. 15 points ____________
2. 18 points ____________
3. 20 points ____________
4. 21 points ____________
5. 26 points ____________

Total (100 points) ____________

1. **[15 points]** (3 points per answer)
In order to discourage guessing, 3 points will be deducted for each incorrect answer (no penalty or gain for blank answers). A net negative score will reduce your total exam score. For each of the following statements, determine if it is true or false for any three events A, B and C in a common probability space.

TRUE	FALSE	
☐	☐	$P(A\|B) + P(A\|B^c) = 1$.
☐	☐	$P(A\|B) + P(C\|B) = 1$.
☐	☐	If A and B are mutually exclusive, then $P(A \cup C\|B) = P(C\|B)$.
☐	☐	If $E_1, \ldots, E_n$ is a partition of Ω, then $\sum_{i=1}^{n} P(E_i\|B) = 1$.
☐	☐	If $E_1, \ldots, E_n$ is a partition of Ω and $P(B) < 1$, then $\sum_{i=1}^{n} P(E_i\|B) < 1$.

2. **[18 points]** Dilbert and Wally are playing a game where Wally flips a coin and Dilbert wins if Heads show, while Wally wins if tails show. The coin is biased with $P\{heads\} = p$, but Dilbert does not know that. After five successive wins by Wally, Dilbert thinks that something is not right, so he decides he should now flip the coin (instead of Wally). Let X be the number of total coin flips (Wally's plus Dilbert's) until Dilbert wins.

   (a) Obtain the pmf of X

   (b) Obtain $E[sin(\pi X)]$.

   (c) Obtain $E[3X^2]$.

3. **[20 points]** A job arriving in a certain cloud computing system must be routed to one of eight servers. Due to the loads, the servers all have different rates. Routing the job to the server with the highest rate would require sampling the rates of all eight servers. Instead, the rates of three randomly selected distinct servers are sampled (all choices being equally likely) and the job is routed to the sampled server with the highest service rate.

   (a) Let $A = \{\text{the job is assigned to the server with the highest service rate}\}$. Find $P(A)$.

   (b) Let $B = \{\text{the job is assigned to one of the four slowest servers}\}$. Find $P(B)$.

   (c) Let $C = \{\text{job is assigned to one of the two servers with the highest rates}\}$. Find $P(C)$.

4. **[21 points]** Consider a binary hypothesis testing problem where X is Geometric(1/2) under hypothesis H_1 and $Geometric(2/3)$ under hypothesis H_0. It is known that $\pi_1 = 4/9$.

   (a) Derive the MAP rule to decide on the underlying hypothesis.

   (b) Determines the probability of false alarm for the MAP rule.

   (c) Determine the value(s) of π_1 for which H_1 would always be chosen.

5. **[26 points]** Alice and Dilbert play the following game of dice. Each round of the game consists of Alice rolling one die and Dilbert rolling two dice. All the dice are fair. Here are the rules of this game:

- If Alice's number falls strictly between the two numbers rolled by Dilbert, then Alice wins and the game ends, e.g. if Alice rolls a 3 and Dilbert rolls a 1 and a 4.
- If Alice's number is equal to one of the numbers rolled by Dilbert, then it is a tie and the round is repeated until there is a winner, e.g if Alice rolls a 3 and Dilbert rolls a 1 and a 3.
- In all other cases, Dilbert wins and the game ends, e.g if Alice rolls a 3 and Dilbert rolls a 1 and a 2.

Based on these rules, obtain:

(a) $P\{\text{Alice wins on the first round given that she rolls a 3}\}$.

(b) $P\{\text{Alice wins on the first round}\}$.

(c) $P\{\text{There is a tie on the first round}\}$.

(d) $P\{\text{Dilbert wins on the first round.}\}$

(e) $P\{\text{Alice wins the game}\}$.