ECE 313: Midterm I

Name: (in BLOCK CAPITALS) _____

Net ID: _____

University ID Number: _____

Signature: _____

Instructions

This exam is closed book and closed notes except that one 8.5"×11" page of notes is permitted: only one side of a sheet is allowed. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

Write your answers in the boxes provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75). It is OK for your final answers to include terms like $\binom{100}{20}, \binom{a}{10}, a^k - b^{k-1}, 2^{50}$, and so on.

SHOW YOUR WORK. answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page.

1. 30 points _____

2. 18 points _____

3. 18 points _____

4. 16 points _____

5. 18 points _____

Total (100 points) _____

1. [**30 points**] True or False? A statement is True if and only if it is always true. Each correct choice counts +3 points, whereas an incorrect choice counts -1 point.

   (a) True ☐   False ☐
   For two events A and B , $P\left(A \cup B\right) \leq P\left(A\right) + P\left(B\right)$.

   (b) True ☐   False ☐
   For any two events A and B which are independent, $P\left(A \cup B\right) = P\left(A\right) + P\left(B\right)$.

   (c) True ☐   False ☐
   If A,B, and C are independent events, then A is independent of $B \backslash C$. (For two sets S and T, $S \backslash T$ is defined is $S \cap T^c$)

   (d) True ☐   False ☐
   For any random variables X and Y , we have $E\left[X + Y^2\right] = E\left[X\right] + \left(E\left[Y\right]\right)^2$.

   (e) True ☐   False ☐
   The following is a valid pmf for a random variable X .

   $$p_X\left(i\right) = \left(e - 1\right) e^{-i} \qquad \text{for } i = 1, 2, \cdots$$

   (f) True ☐   False ☐
   For any random variables X , we have $Var(2X + 1) = 2Var(X) + 1$.

   (g) True ☐   False ☐
   Let A and B be events with probabilities $P\left(A\right) = \frac{3}{4}$ and $P\left(B\right) = \frac{2}{3}$. It is possible to have $P\left(A \cap B\right) = \frac{1}{3}$.

   (h) True ☐   False ☐
   For sets A, B, and C , $A \backslash \left(B \cap C^c\right) = \left(A \backslash B\right) \cup \left(A \backslash C^c\right)$.

   (i) True ☐   False ☐
   Let X be a geometric random variable with parameter $0 < p < 1$. The pmf $p_X\left(k\right)$ obtains its maximum at $\lfloor 1/p \rfloor$.

   (j) True ☐   False ☐
   If $P(A) = 0.3, P(B) = 0.7, P(B|A) = 0.5$, then $P(A^c B^c) = 0.1$.

2. [**18 points**] A standard deck of 52 cards contains 4 aces. Suppose we shuffle all cards randomly so that all 52! permutations being equally likely. Compute the following:

(a) [**6 points**] The probability that all of the top 4 cards in the deck are aces.

$ANS =$

(b) [**6 points**] The probability that none of the top 4 cards in the deck are aces.

$ANS =$

PROBLEM 2, CONTINUED

(c) [**6 points**] The conditional probaility that "none of the top 4 cards in the deck are aces, given that the suit of the top card is SPADE (♠)".

$ANS =$

3. **[18 points]** A fair die is rolled 5 times, let X be the largest die value. For example, if the outcome is $(1, 2, 5, 4, 3)$ then $X = 5$:

(a) **[5 points]** Find $P(X \leq 3)$.

$$ANS =$$

(b) **[7 points]** Find $P(X = 4)$.

$$ANS =$$

PROBLEM 3, CONTINUED

(c) [**6 points**] Find the probability that exactly two die values are LARGER than 4 and all other three die values are LESS THAN OR EQUAL TO 4 .

$ANS =$

4. **[16 points]** ECE department has approximately 2000 students; 40% female and 60% male. Among these 2000 students there are 300 international students, 20% of whom are female. A student is chosen at random from ECE.

(a) **[7 points]** What is the conditional probability that the student is male given that the student is NOT an international student?

$ANS =$

(b) **[7 points]** What is the conditional probability that the student is an international student given that the student is female?

$ANS =$

(c) **[2 points]** True □   False □   The two events are independent: {The student is male}, {The student is international}. (Incorrect choice counts -1 point.)

6

5. **[18 points]** A fair die is rolled until the numbers $2, 4$ and 6 show up (not in any particular order or in succession). Let M be the number of rolls.

(a) **[5 points]** Find $P(M = 3)$.

ANS =

(b) **[5 points]** Find $E[M]$

ANS =

PROBLEM 5, CONTINUED

(c) [**8 points**] Find $P(M = 4)$.

$ANS =$