

Assigned: Thursday, July 3

Due: Problems 1-4 Monday, July 7; Problem 5-10 Thursday, July 10

Reading: Ross Chapters 3.1-3.5

Noncredit Exercises: (Do not turn these in) Ross Chapter 3: Problems 1,2,5,10,31,38,44,53,62,72.

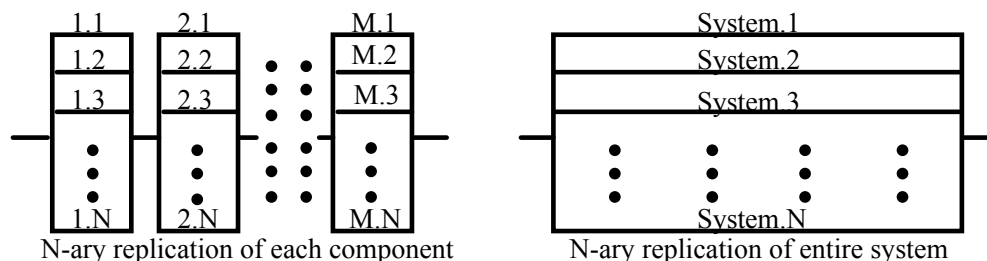
1. Ross, Problem 5, p. 120 (6th Edition), or p.123 (5th Edition). In addition to the probability asked for, find the probability that the second ball is red, and determine if it is smaller or larger or the same as the probability that the first ball is red.
2. The experiment consists of picking a flight at random from all the United Airlines and America West flights landing at Chicago, Los Angeles, Phoenix, San Diego, or San Francisco. Let U and W respectively denote the event that the chosen flight is an United Airlines or an America West flight, let C, L, X, D , and F respectively denote the event that the chosen flight is landing at Chicago, Los Angeles, Phoenix, San Diego, or San Francisco, and let T denote the event that the chosen flight is on time. The conditional probabilities of on-time arrival are as follows:
 $P(T|UC) = 0.85, \quad P(T|UL) = 0.92, \quad P(T|UX) = 0.95, \quad P(T|UD) = 0.91, \quad P(T|UF) = 0.83,$
 $P(T|WC) = 0.78, \quad P(T|WL) = 0.88, \quad P(T|WX) = 0.92, \quad P(T|WD) = 0.85 \quad P(T|WF) = 0.73.$
 - (a) Based on this data, which airline would you say has better on-time performance? Does the answer depend on which airport you are talking about?
 - (b) Use the fact that $\{C, L, X, D, F\}$ form a partition of the sample space to show that the average on-time arrival probability $P(T|U)$ for United flights is given by

$$P(T|U) = P(T|UC)P(C|U) + P(T|UL)P(L|U) + P(T|UX)P(X|U) + P(T|UD)P(D|U) + P(T|UF)P(F|U)$$
where $P(C|U)$ is the conditional probability that the flight is landing at Chicago given that it is a United flight etc. State a similar expression for $P(T|W)$.
 - (c) 60% of United Airlines flights land at its hub (snowy Chicago), 15% at each of LA and San Francisco, and 5% at each of Phoenix and San Diego. 75% of America West flights land at its hub (sunny Phoenix), 10% at LA, and 5% at each of the other three airports. Use these numbers in the formula of part (b) and show that $P(T|U) < P(T|W)$, i.e., United has a worse average on-time performance even though it beats America West at all the five airports! Write a short explanation of the discrepancy between the per-airport on-time performance and the overall on-time performance.
3. Each box of Cornies, the breakfast of silver medalists, contains one picture, which is of Luke Skywalker with probability $2/3$ and of Darth Vader with probability $1/3$, independently of which picture is in any other box of Cornies. Little Jimmy Kirk of Cedar Rapids, Iowa, asks his mother to buy boxes of Cornies until he has at least one picture of both beings, and his mother agrees to do so.
 - (a) What is the minimum number of boxes of Cornies that Mrs Kirk must buy?
 - (b) Let X denote the number of boxes of Cornies Mrs Kirk purchases until such time as Jimmy has acquired at least one picture of each of the two entities. What is the pmf of X ? Verify that the total probability mass specified by your pmf does equal 1.
 - (c) What is the conditional pmf of X given that the first box contained a picture of Luke? What is the conditional pmf of X given that the first box contained a picture of Darth?
 - (d) Use the theorem of total probability to compute the unconditional pmf of X from the conditional pmfs found in part (c). Do you get the same answer as in part (b)? Why not?

- (e) Find the mean and variance of X . [Hint: it might be easier to determine the conditional means and variances and then combine them to obtain the unconditional mean and variance]
4. Two of the eleven letters in a road sign that reads CHATTANOOGA have fallen down. Assume that each pair of letters is equally likely to have fallen down. A drunk randomly puts the fallen letters back into the two empty slots, possibly interchanging the positions of the letters, and possibly putting the letters back upside down. Thus, all eight possibilities corresponding to the three binary choices
- {letters put back in correct position or interchanged}
 {left-hand letter upside down or rightside up}
 {right-hand letter upside down or rightside up}
- are equally likely.
- (a) What is the probability that the sign still **reads** CHATTANOOGA?
- (b) What is the probability that all the letters **seem** to be correct side up but the sign does not read CHATTANOOGA?
- (c) What is the probability that only one letter **seems** to be upside down?
- (d) What is the probability that two letters **seem** to be upside down?
- (e) Given that all the letters appear to be right side up, what is the (conditional) probability that **at least one** vowel fell down?
- (f) A designated driver, who knows that the sign is supposed to read CHATTANOOGA, observes the restored sign. What is the probability that the driver can **correctly** identify (without any possibility of making a mistake) which letters had fallen down?
5. The dice game of craps begins with the player (called the shooter) rolling two fair dice. If the result is a 2, or 3, or 12, the shooter loses, while if the result is a 7 or 11, the shooter wins. The shooter who rolls any of 4, 5, 6, 8, 9, 10 has neither won nor lost (as yet). What happens then is discussed in parts (b) and (c).
- (a) What is the probability that the shooter loses on the first roll? What is the probability that the shooter wins on the first roll?
- (b) If the sum of the dice on the **first roll** is any of 4, 5, 6, 8, 9, 10, that number is called the **shooter's point**. For **each** number i in the set $\{4, 5, 6, 8, 9, 10\}$, find the probability that the shooter's point is i . I need six answers here, folks!
- (c) Suppose that the shooter's point is i where i is some number in $\{4, 5, 6, 8, 9, 10\}$. The shooter now rolls the two dice again. If the result is a 7, the shooter loses (craps out.) If the result is i , the shooter wins (this is referred to as making the point). If the result is neither i nor 7, the shooter rolls again. This process continues until the shooter either makes the point or craps out. Given that the shooter's point is i , what is the conditional probability that the shooter makes the point? Naturally, the answer depends on i , so here too, I need six answers.
- (d) Use the above results to compute the probability of winning at craps.
- (e) Given that the shooter's point is 8, what is the probability that the shooter makes it "the hard way," that is, by rolling two fours? Generally, bets are offered at 10-to-1 odds that the shooter makes the point 8 the hard way. That is, if you bet \$1, you win \$10 (plus your \$1 back!) if the shooter makes 8 the hard way; and you lose the \$1 that you bet if the shooter craps out or makes 8 by rolling 2-6, 3-5, 5-3, or 6-2). In the long run over many such bets, do you expect to make money, or lose money, or come out even?
6. Consider the following simplified model for a game of tennis. On each serve, let p denote the probability that player A wins the point, and $q = 1-p$ the probability that

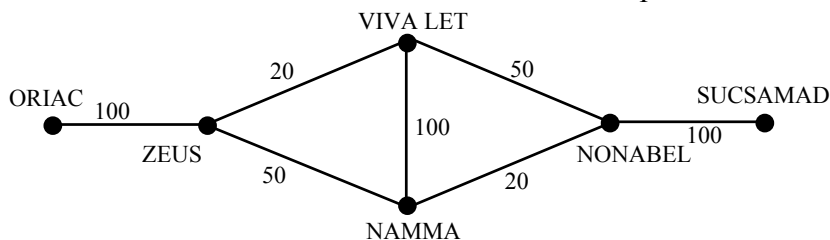
player B wins the point. Assume that the outcome of each serve is independent of all others. Player A wins the game if the score reaches 4–0, 4–1, or 4–2, while B wins the game if the score reaches 2–4, 1–4, or 0–4. Else, the score reaches 3–3 (called deuce) and from this point onwards, the game continues until one player is two points ahead of the other, and thereby wins the game.

- (a) Find the probabilities that the score reaches 4–0, 4–1, or 4–2 and the probabilities that the score reaches 2–4, 1–4, or 0–4. I need 6 answers here!
 - (b) Find $P(\text{score reaches deuce})$. Show that the sum of the seven probabilities obtained in parts (a) and (b) is 1 regardless of the value of p .
 - (c) Given that the score is deuce, what is $P(\text{A wins the next two points})$? (This means A wins the game). What is $P(\text{B wins the next two points})$? (This means B wins the game). What is the probability that both players win one point each? In this case, the score is tied again, and is also called deuce.
 - (d) Once the score reaches deuce, there *may* be further deuces until ultimately, either A or B wins both points and thereby wins the game. What is the probability that A ultimately wins the game given that the score is deuce? What is the probability that B ultimately wins the game given that the score is deuce? (Hint: these answers are different from those of part (c)) What is the probability that the game goes on forever with the score continuing to reach deuce after every two points?
7. The Senate of a certain country has 100 members consisting of 43 Conservative Republicans, 21 Conservative Democrats, 12 Liberal Republicans, and 24 Liberal Democrats. Before each vote, the groups caucus separately. Each group decides *independently* of the other groups whether to support or oppose the motion. *All* members of the group then vote in accordance with the caucus decision.
- (a) Let A, B, C, and D respectively denote the events that the four groups vote for a spending plan that will lead to a 50% increase in the DoD budget over the next two years. Suppose that the probabilities of these independent events are $P(A) = 0.9$, $P(B) = 0.6$, $P(C) = 0.5$ and $P(D) = 0.2$. What is the probability that the bill passes?
 - (b) The President vetoes the bill as a budget-breaker. Let E, F, G, and H respectively denote the independent events that the four groups support the motion to override the veto. If these events have probabilities $P(E) = 0.99$, $P(F) = 0.4$, $P(G) = 0.6$, and $P(H) = 0.1$, what is the probability that the motion to override the veto passes ?
- Political innocents are reminded that a simple majority (51 or more votes) is required to pass a bill, and a two-thirds majority (67 or more votes) to override a veto.
8. A system works if and only if all of its M components (numbered 1 through M) work. Each component fails (independently) with probability p . Consider two possible means of obtaining a more reliable system. We can replicate each component N times as shown in the graph model on the left. Or, we can replicate the entire system N times as shown in the graph model on the right. In either case, the result is called a replicated system. Note that both methods use the *same* number N of each component .



As a more concrete example of the question, consider which of the following two methods provides more reliable transportation

- a single gigantic car with N engines, N transmissions, N brakes, ... etc. that works (i.e. provides us with transportation) as long as at least one of its engines and at least one of its transmissions, and at least one of its brakes ... works.
 - N separate ordinary cars that fail as soon as any one of their components fail, but which together provide us with transportation as long as at least one car is in working condition.
- (a) For each model, find the probability of replicated system failure in terms of p , N and M .
- (b) Suppose that $M = 5$ and $p = 0.2$. If it is desired that the replicated system failure probability be less than 0.001, what should N be in each case?
- (c) Repeat part (b) assuming that $M = 1000$.
9. MiddleEast Bell, a division of Psingular Corp., has built a telephone network as shown below. Terrorists attack each of the seven links. The attacks may be considered to be independent events, and the attack on a link succeeds in severing the link with probability p . If a link is severed, switches automatically re-route calls so as to avoid the failed link (if possible).
- (a) What is the probability of being able to call from ORIAC to SUCSAMAD?
- (b) Given that it is possible to call from ORIAC to SUCSAMAD, what is the conditional probability that the ZEUS to NAMMA link is in working condition?
- (c) The link capacities (i.e., the numbers of telephone calls that the links can carry (in either direction)) are as marked on the diagram. Let X denote the number of calls that can be made from ORIAC to SUCSAMAD. Find the pmf and the expected value of X .



10. A QMR (quintuple modular redundancy) system is a fancier and more expensive version of the TMR system studied in class. It uses 5 identical circuits.
- (a) If each circuit has probability p of failing, what is the probability that the majority gate output is incorrect? Ignore the possibility that the majority gate has failed. (Hint: condition on IV and V both failed, one of IV and V failed, and neither IV nor V failed; combine results using the theorem of total probability)
- (b) A graph model for the TMR system was discussed in class where it was shown that we must replicate links, e.g. each circuit is represented by more than one link, and if the circuit fails, all these links are removed from the graph. Consider the graph model of the QMR system. If there are no failures, how many paths are there from In to Out? How many links represent each of the circuits?