

Assigned: Monday, June 16

Due: Thursday, June 19

Reading: Ross, Chapter 1.1–1.5, Chapter 2.1–2.5 and 2.7

Problems: Most of the topics covered in this problem set will be needed *after* the drop date.

Hence, this problem set (which is based entirely on material covered in the *prerequisites* to this course) is intended as a review and as a diagnostic aid for identifying areas that you may need to review one more time before starting the course.

- 1.(a) Prove that $1 + x + x^2 + \dots + x^{n-1} = \frac{1 - x^n}{1 - x}$ for all $x \neq 1$ and all integers $n \geq 1$.
- (b) Assuming $|x| < 1$, find the sum $1 + x + x^2 + \dots$ (By definition, this sum is the limit of the sum $1 + x + x^2 + \dots + x^{n-1}$ as n goes to infinity)
- (c) Compute the first four terms of the MacLaurin series (Taylor series in the vicinity of 0) for $f(x) = (1 + x)^n$. These terms constitute the 3rd degree Taylor polynomial for $f(x)$.
- (d) Repeat part (c) for $g(x) = (1 - x)^n$.
- (e) True or false? The Taylor *series* for $(1 + x)^n + (1 - x)^n$ contains only the even powers of x

- 2.(a) Find the limit of $\frac{1}{[\sin x]^2} - \frac{1}{x^2}$ as x approaches 0. Use your calculator to evaluate this function for *small* values of x say, $x = 10^{-1}$, $x = 10^{-2}$, $x = 10^{-3}$, etc. Does the function seem to be approaching a limit, and if so, what do you think is the limit? Now, use what you have learned about limits in calculus to find $\lim_{x \rightarrow 0} \frac{1}{[\sin x]^2} - \frac{1}{x^2}$ analytically.
(Hint: the answer is not 0, or 1, or ∞).
- (b) Find all the maxima (maxima = plural of maximum) of $f(x) = x^4(1.001)^{-x^2}$ on the real line.

3. Find the values of the following integrals

(a) $\int_{-2}^1 |x| \, dx$

(b) $\int_0^1 x(1-x)^{19} \, dx$

(c) $\int_0^1 x \exp(-x) \, dx$

4. Let $\frac{d}{dx} f(x) = g(x)$ for $-\infty < x < \infty$. Which of the following statements are true for all x , $-\infty < x < \infty$? (Here, C denotes an arbitrary constant.)

(i) $\frac{d}{dx} f(-x) = -g(-x)$. (ii) $\frac{d}{dx} f(x^2) = x g(x^2)$.

(iii) $\int g(-x) dx = -f(-x) + C$. (iv) $\int g(x^2) dx = f(x^2)/(2x)$

- 5.(a) Integrate $f(x, y) = \max(x, y)$, over the region $\{(x, y) : 0 < x < 2, 0 < y < 1\}$.
- (b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x, y) : x^2 + y^2 > 2\}$.