ECE 313
FINAL EXAMINATION

Thursday August 7, 2003
8:00 a.m. - 10:00 a.m.
170 Everitt Laboratory

Name ______________________________ University Net-ID_____________________
Print in BLOCK CAPITALS, **not** *cursive script*

Signature ___________________________

INSTRUCTIONS

This exam consists of 6 Problems worth a total of 225 points. Note that the problems carry unequal credit, and pace yourself accordingly.

This exam is closed book; however, both sides of two $8^1/_2$" by 11" sheets of notes may be consulted. Calculators, laptop computers, Palm Pilots, cellular phones, wireless two-way e-mail pagers, tables of integrals, etc. may not be used. The last page of the exam is a tear-off sheet with the phi function table.

Write your answers in the spaces provided (except for the last two problems). *Some answers can be given in terms of common constants (e.g.,* π, e, $\sqrt{2}$ *etc) or functions (e.g.* exp, ln, sin, cos, *etc.) but the answer should be simplified as much as possible.* Common fractions (such as 5/16) are preferable to decimal fractions (such as 0.3125), but simplify the fractions by cancelling common factors from the numerator and denominator (for example, write 5/16 instead of 15/48).

Please show all your work. If you need extra space, use the back of the previous page. *Answers without appropriate justification will receive no credit.*

SCORES

1.	30	points	______
2.	40	points	______
3.	45	points	______
4.	35	points	______
5.	45	points	______
6.	30	points	______
TOTAL	(max 225 points)		______

This exam booklet consists of this cover-sheet, 7 additional pages containing 6 problems, and a tear-off page with a table of Φ(x).

1. **(30 points)** There is a collection of basketball players, which can be divided into two groups 1 and 2. Group 1 has 3, Group 2 has 6 players. Players are tested on their abilities of shooting free throws. A player from Group 1 can make a free throw with probability 1/2, whereas a player chosen from Group 2, can make a free throw with probability 1/3, independent of the outcome of any other free throws by himself (herself) or any other player.

(a) **(10 points)** A player is selected at random from among all the players and attempts one free throw. Let A be the event that the free throw is made. Find P[A].

(b) **(20 points)** A player is selected at random and attempts 10 free throws. Let **M** be the random variable denoting the number of misses. Determine P[**M**=4].

P[A] =
P[**M**=4] =

2. **(40 points)** Automated manufacturing of TV sets in a factory is done by passing parts through a sequence of machines. The factory has four machines, M_1, M_2, M_3, M_4. Each of the four machines fail independently with probability 1– p.

A TV set can be manufactured using any one of two sequences:

(I) M_1, M_2 , or **(II)** M_3, M_4, M_2 (in that order)

(a) **(20 points)** Find the probability that the factory can manufacture TV sets.

(b) **(20 points)** Find the probability that M_4 has failed, given that the factory can manufacture TV sets.

P(TV sets can be manufactured) =
P(M_4 failed | TV sets can be manufactured) =

3. **(45 points)** Let **X** be a Gaussian random variable with mean 1 and variance 4.

(a) **(20 points)** Find the probability that the roots of the quadratic equation
$4z^2 + 4(\mathbf{X}+1)z + 7\mathbf{X} - 5 = 0$
are real.

(b) **(25 points)** Find the cumulative distribution function (CDF) of the random variable
$\mathbf{Y} = max(\mathbf{X},5) - min(\mathbf{X},5)$

Remark: In part (a), use the Φ function table provided to find the probability numerically. In part (b) express your answer in terms of Φ(x), the distribution function of the unit Gaussian random variable.

P{real roots} =
$F_{\mathbf{Y}}(v) = \begin{cases} \\ \end{cases}$

4. **(35 points) X** and **Y** are *independent* discrete random variables.

X is a geometric random variable with parameter p, $0 < p < 1$.

Y = 1 + **Z** where **Z** is a Poisson random variable with parameter λ where $\lambda > 0$.

(b) **(20 points)** For $k \geq 1$, write an expression for P{**X** = k, **Y** = k} in terms of k, p, and λ.

(c) **(15 points)** Find the probability that **X** equals **Y**. Express your answer in the simplest possible terms

P{**X** = k, **Y** = k} =
P{**X** = **Y**} =

5. **(45 points)** **X** and **Y** are are continuous random variables with joint PDF

$$f_{\mathbf{X,Y}}(u,v) = \begin{cases} 4e^{-2v}, & 0 \le u < v \\ 0, & otherwise \end{cases}$$

(a) **(10 points)** Find the marginal PDF of **X**, $f_{\mathbf{X}}(u)$.

(b) **(10 points)** Find the conditional PDF of **Y** given **X**, $f_{\mathbf{Y|X}}(v|u)$.

(c) **(25 points)** What is the PDF of $\mathbf{Z} = \mathbf{Y} - \mathbf{X}$?

6. **(30 points)** A signal, **X,** is transmitted from a temperature sensor on a communication satellite is a Gaussian random variable with E[**X**] = 0, and Var[**X**]=9. The receiver at mission control receives **Y** = **X** + **N**, where **N** is an *independent* noise voltage with a PDF

$$f_{\mathbf{N}}(\alpha) = \begin{cases} 1/6, & -3 \le \alpha \le 3 \\ 0, & otherwise \end{cases}$$

Note that E[**N**] = 0, and Var[**N**] = 3. The receiver uses the observed value of **Y** = v to calculate a linear estimate of the signal **X**,

$$\hat{X}(v) = av + b$$

(a) **(25 points)** Whate are the optimum mean square values of a and b in the linear estimator (LMMSE)?

(b) **(5 points)** What is the mimimum mean square error of the linear estimator?

Φ(x), the area under the standard Gaussian density function to the left of x

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998