1. **TRUE** $F_X(b)$ is right continuous.
FALSE It is possible that $F_X(a) = F_X(b)$.
TRUE This is the certain event.
FALSE This need not hold.
TRUE This is true, since **X** is a continuous random variable.
TRUE This has to hold for all density functions.

2. $P\{|\mathbf{X} - 4| > 3\} = P\{\mathbf{X} > 7\} + P\{\mathbf{X} < 1\} = 1 - \Phi\left(\frac{7-2}{4}\right) + \Phi\left(\frac{1-2}{4}\right) = 1 - \Phi(1.25) + \Phi(-0.25)$

$= 1 - \Phi(1.25) + 1 - \Phi(0.25) = 2 - 0.8944 - 0.5987 = 0.5069.$

$P\{\mathbf{X} < 3 \mid \mathbf{X} > 2\} = P\{2 < \mathbf{X} < 3\}/P\{\mathbf{X} > 2\} = 2(\Phi(0.25) - \Phi(0)) = 2\,\Phi(0.25) - 1 = 1.1974 - 1 = 0.1974$

3.(a) The PDF of **X** has value 0.2 for $-1 \le u \le 4$. Using LOTUS we have

$$E[\mathbf{Y}] = E[|\mathbf{X}-1|] = \int_{-1}^{1} 0.2(1-u)du + \int_{1}^{4} 0.2(u-1)du = 0.2(u - u^2/2)|_{-1}^{1} + 0.2(u^2/2 - u)|_{1}^{4} = 1.3\,.$$

(b) **Y** takes on values in the range [0,3]. Thus, for $v < 0$ or $v > 3$, $F_Y(v) = 0$. For any v, $0 \le v \le 2$,
$F_Y(v) = P[\mathbf{Y} \le v] = P[1 - v \le \mathbf{X} \le v + 1] = 0.2(v + 1 - (1 - v)) = 0.4v$, while for any v, $2 \le v \le 3$,
$F_Y(v) = P[\mathbf{Y} \le v] = P[-1 \le \mathbf{X} \le v + 1] = 0.2(v + 1 - (-1)) = 0.2(v+2)$. Differentiating, we obtain

$$f_Y(v) = \begin{cases} 0.4, 0 \le v \le 2 \\ 0.2, 2 \le v \le 3 \\ 0, elsewhere \end{cases}$$

4. Let $\mu = 2$ denote the arrival rate of the process. Then, both **N**(0,T] and **N**(0.5T, 1.5T] are Poisson random variables with parameter $\mu T = 2T$.

(a) Hence, $P(A) = P\{\mathbf{N}(0,T] = 0\} = \exp(-2T)$ and $P(B) = P\{\mathbf{N}(0.5T, 1.5T] = 1\} = 2T \cdot \exp(-2T)$.

(b) $P(AB) = P\{\mathbf{N}(0,T] = 0, \mathbf{N}(0.5T, 1.5T] = 1\} = P\{\mathbf{N}(0,0.5T] = 0, \mathbf{N}(0.5T, 1.5T] = 1\}$ since the single arrival must have occurred during (0.5T, 1.5T]. But, **N**(0,0.5T] and **N**(0.5T, 1.5T] are independent random variables because the time intervals are disjoint. Hence,
$P(AB) = P\{\mathbf{N}(0,0.5T] = 0, \mathbf{N}(0.5T, 1.5T] = 1\} = P\{\mathbf{N}(0,0.5T] = 0\} \cdot P\mathbf{N}(0.5T, 1.5T] = 1\}$
$= (2T/2) \cdot \exp(-2T/2) \cdot \exp(-2T) = T \cdot \exp(-3T)$, and $P(B|A) = P(AB)/P(A) = T \cdot \exp(-T)$.

5. Since X is uniform its density function is

$$f_X(p) = \begin{cases} 5, 0.4 \le p \le 0.6 \\ 0, elsewhere \end{cases}$$

Let A be the event A={1 head in 1 toss}. We know $P[A \mid \mathbf{X} = p] = p$. The posterior density is

$$f_{X|A}(p) = \frac{P[A \mid X = p] f_X(p)}{\int_{0.4}^{0.6} P[A \mid X = p] f_X(p)}$$. Note the limits.

Carrying out the integral we obtain

$$f_{X|A}(p) = \begin{cases} 10p, 0.4 \le p \le 0.6 \\ 0, elsewhere \end{cases}$$

The Bayesian estimate of *heads* at the next tossing is just

$$P\{\text{Heads}\} = \int_{0.4}^{0.6} p f_{X|A}(p) dp = \int_{0.4}^{0.6} p 10 p dp = \frac{10}{3} p^3 |_{0.4}^{0.6} = \frac{10}{3}(0.216 - 0.064) = \frac{1.52}{3} = 0.5067 > 0.5$$

Note that, based on the observation our estimate of the probability of observing heads at the next tossing has increased.