

1. Check the appropriate box for each statement below. Answers need not be justified. However, in order to discourage guessing, you will be penalized for wrong answers.

Which of the following statements are true for all CDFs?

TRUE	FALSE	
<input type="checkbox"/>	<input type="checkbox"/>	$P\{X > b\} = 1 - F_X(b)$
<input type="checkbox"/>	<input type="checkbox"/>	If $a < b$ , then $F_X(a) < F_X(b)$
<input type="checkbox"/>	<input type="checkbox"/>	$\lim_{a \rightarrow \infty} F_X(a) = 1$

Let  $f_X(u)$  denote the probability density function (PDF) of a *continuous* random variable  $X$ .

Which of the following statements are true for all PDFs?

TRUE	FALSE	
<input type="checkbox"/>	<input type="checkbox"/>	$f_X(u) \leq 1$ for all $u$ , $-\infty < u < \infty$
<input type="checkbox"/>	<input type="checkbox"/>	$P\{a < X < b\} = P\{a < X \leq b\}$
<input type="checkbox"/>	<input type="checkbox"/>	$\lim_{u \rightarrow -\infty} f_X(u) = 0$

2.  $X$  denotes a Gaussian random variable with mean 2 and variance 16. Find

$P\{|X - 4| > 3\}$  and  $P\{X < 3 \mid X > 2\}$  using the table of values of the unit Gaussian CDF  $\Phi(\bullet)$ .

3. Let  $X$  denote a random variable uniformly distributed on the interval  $[-1, 4]$ , and

let  $Y = |X - 1|$

(a) Find  $E[Y]$ .

(b) Find  $f_Y(v)$ , the probability density function of  $Y$ .

4. Consider a Poisson process with arrival rate 2 per second. Let  $A$  denote the event that there are *no arrivals* in the time interval  $(0, T]$  and  $B$  the event that there is *exactly one* arrival in the time interval  $(0.5T, 1.5T]$ .

(a) What are the values of  $P(A)$  and  $P(B)$ ?

(b) Find  $P(B \mid A)$ .

5. A coin is tossed once, and *heads* shows. Assuming that the probability  $p$  of *heads* is the value of a random variable  $X$  uniformly distributed in the interval  $(0.4, 0.6)$ , find the probability that at the next tossing *heads* will show. (**Hint:** The asked probability is the Bayesian estimate of probability of *heads* after the observation).