

1. (22 points) Check the appropriate box for each statement below. Answers need not be justified. However, in order to discourage guessing, you will be penalized for wrong answers.

(a) (10 points) (+2 points for a correct answer, -2 points for a wrong answer, and 0 points for no answer) Which of the following statements are true for all events A and B such that $0 < P(A) < 1$, $0 < P(B) < 1$?

TRUE FALSE

- | | | |
|--------------------------|--------------------------|--|
| <input type="checkbox"/> | <input type="checkbox"/> | $P(A B) > P(A)$ |
| <input type="checkbox"/> | <input type="checkbox"/> | $P(A B) + P(A B^c) = 1$ |
| <input type="checkbox"/> | <input type="checkbox"/> | $P(A B) + P(A^c B) = 1$ |
| <input type="checkbox"/> | <input type="checkbox"/> | $P(B A) P(A) + P(B A^c) P(A^c) = P(B)$ |
| <input type="checkbox"/> | <input type="checkbox"/> | $P(A B) = P(B A) P(B) / P(A)$ |

(b) (12 points) (+12 points for a correct answer, -3 points for a wrong answer, and 0 points for no answer) Which of the following four statements are true for all events A and B such that $0 < P(A) < 1$, $0 < P(B) < 1$?

- | | |
|---|--|
| (a) $P(A \cap B) \leq \min\{P(A), P(B)\}$ | (b) $P(A \cup B) \geq [P(A) + P(B)] / 2$ |
| (c) $P(A \cap B) \geq P(A) + P(B) - 1$ | (d) $P(A \cap B) \leq P(A B)$ |

<input type="checkbox"/> Only (a) and (c) are true statements.
<input type="checkbox"/> Only (b) and (c) are true statements.
<input type="checkbox"/> Only (a), (b), and (c) are true statements.
<input type="checkbox"/> All four are true statements.
<input type="checkbox"/> None of the above: only the following are true statements_____

2. (24 points) A certain town has three newspapers A, B, and C. The proportions of townspeople that read these newspapers are as follows:

A: 10%, B: 30%, C: 5%; A and B: 8%, A and C: 2%, B and C: 4%, and 1% read all three newspapers.

- (a) (6 points) What percentage of people read only one newspaper?
 (b) (6 points) What percentage of people read at least two newspapers?
 (c) (6 points) If A and C are morning papers and B is an evening paper, what percentage of people read at least one morning paper as well as the evening paper?
 (d) (6 points) How many people do not read any newspapers?

3. **(30 points)** Let X denote the number of Heads observed in 10 tosses of a fair coin.
- (a) **(4 points)** What kind of random variable is X , and what is its average value?
- (b) **(6 points)** Let A be the event that 4th toss is a Head. What is $P(A)$?
- (c) **(6 points)** What is $E[X | A]$?
- (d) **(14 points)** Let B be the event that you observe *no more than 2* Heads. What is $P(A | B)$?
4. **(14 points)** You have 3 coins in your pocket, 2 of which are fair ($P(\text{Heads})=P(\text{Tails})=1/2$) and 1 is biased with $P(\text{Heads})=1/4$. You pick **two** coins out of your pocket at random and toss each one **two** times. Let X be the random variable denoting the **total** number of Heads you observe. What is $P(X = 2)$?
5. **(10 points)** Let X denote a geometric random variable with parameter $1/5$. What is the average value of $(X-1)^2$ given that $X \leq 2$?