University of Illinois **Spring 2026**

ECE 313: Problem Set 2

Due: Friday, February 6 at 06:59:00 p.m.
Reading: *ECE 313 Course Notes, Section 1.2, 2.1, 2.2*

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Discrete pmf with parameters]**
   Suppose that the random variable X has the following pmf:

$$p_X(k) = \begin{cases} c_1 \left(\frac{1}{2}\right)^k, & \text{if } k = -2, -1, 0, \\ c_2, & \text{if } k = m, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Given that $p_X(m) = 5p_X(0)$, what are the values of the constants c_1 and c_2 so that we have a valid pmf?
   (b) Find the value of m such that $E[X] = 0$, using the constants c_1, c_2 found in part (a).
   (c) With the constants c_1, c_2 found in part (a), and the value of m found in part (b), find the variance of the random variable X, i.e., $\text{Var}(X)$.
   (d) Find $P(|X| \geq 2)$ and $E[|X|]$, for the values of the constants c_1, c_2 found in part (a) and the value of m found in part (b).

2. **[Mean, Variance, LOTUS]**
   Consider a random variable X.

   (a) Is it possible for the mean of X to be 10 and the standard deviation 0? If so, construct a pmf for X that corresponds to these values. If not, prove the impossibility.
   (b) Is it possible for the mean of X to be 0, the second moment $E[X^2] = 1$, and the standard deviation 1? If so, construct a pmf for X that corresponds to these values. If not, prove the impossibility.

(c) Suppose the mean of X is μ and the standard deviation is σ. Find a formula for the mean of the random variable $Y = 2X^2 + 6X - 1$ in terms of μ and σ.

3. **[Gambling with Dice]**
   You roll a fair die. If you roll an even number i, you win i dollars. If you roll an odd number, you lose m dollars. Let X denote the amount of money you win (a negative amount indicates a loss).

   (a) Find the pmf of the random variable X. Note that the pmf will depend on m.

   (b) If $E[X] = 0$, find m.

   (c) Fixing the value of m to the value you found in part (b), find $\text{Var}(X)$.

4. **[PMF, Mean and standard deviation]**
   Suppose two fair dice are rolled independently, so the sample space is $\Omega = \{(i, j) : 1 \leq i \leq 6, \text{ and } 1 \leq j \leq 6\}$, and all outcomes are equally likely. Let X be the random variable defined by $X(i, j) = i - j$, and let Y be the random variable defined by $Y(i, j) = \max\{0, i - j\}$ (Rectified Linear Unit or ReLU).

   (a) Derive the pmf of X and sketch it.

   (b) Find the mean, $E[X]$, and standard deviation, σ_X, of X.

   (c) Derive the pmf of Y and sketch it.

   (d) Find the mean, $E[Y]$, and standard deviation, σ_Y, of Y.

5. **[Possible probability assignments]**
   A random experiment has a sample space $\Omega = \{a, b, c, d\}$. Suppose that $P(\{b, c, d\}) = \frac{5}{6}$, $P(\{a, b\}) = \frac{5}{12}$, and $P(\{b, c\}) = \frac{1}{2}$. Use the axioms of probability to find the probabilities of the elementary events ($P(\{a\})$, $P(\{b\})$, $P(\{c\})$, and $P(\{d\})$).