University of Illinois **Spring 2025**

ECE 313: Problem Set 8

Due: Friday, April 4 at 7:00:00 p.m.
Reading: *ECE 313 Course Notes,* Section 3.7 - 3.8

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. [**ML Parameter Estimation**]
   Calls arrive in a call center according to a Poisson process with arrival rate λ (calls/hour). Derive the maximum likelihood estimate $\hat{\lambda}_{ML}$ if k calls are received from 1:00 PM to 1:15 PM.

2. [**Function of a Uniform Random Variable**]
   Let X be a uniform random variable with mean m and variance $\frac{4m^2}{3}$, where m is a positive constant. Find the pdf of $Y = |X|$.

3. [**Function of Gaussian Random Variable**]
   Suppose X is a Gaussian random variable with mean 0 and variance 4.

   (a) Let $Y = \alpha$ if $X > 0$ and $Y = -\alpha$ otherwise. Does Y have a pmf or pdf? What is the pmf or pdf of Y?

   (b) The squared error in representing X by Y (as defined in (a)) is defined as the random variable $Z = \begin{cases} (X-\alpha)^2, & \text{if } X > 0, \\ (X+\alpha)^2, & \text{if } X \leq 0 \end{cases}$. Find the value of α to minimize $\mathrm{E}[Z]$.

   (c) Let W be the value in $\{-4, -2, 0, 2, 4\}$ that is closest to X. That is, $W = -4$ if $X < -3$, $W = -2$ if $-3 \leq X < -1$, etc. Find the pmf (or pdf) of W. Express your answer in terms of Q-functions if necessary.

4. [**Function of Laplacian Random Variable**]
   Suppose X is a random variable with pdf $f_X(x) = \frac{1}{4}e^{-|x|/2},\ x \in \mathbb{R}$.

(a) Solve 3(a) for this new definition of X.

(b) Solve 3(b) for this new definition of X. (**Hint**: Using integration by parts, $\int x^2 e^{-\frac{x}{2}} dx = -e^{-\frac{x}{2}}(2x^2 + 8x + 16) + C$, and $\int x e^{-\frac{x}{2}} dx = -2e^{-\frac{x}{2}}(x+2) + C$)

5. **[Generating a Random Variable]**
Find a function g such that if U is uniformly distributed over $[0, 1]$, $Y = g(U)$ has the following pdf:

$$f_Y(v) = \begin{cases} -\frac{v+2}{4}, & \text{if } -4 \leq v < -2, \\ \frac{v+2}{4}, & \text{if } -2 \leq v < 0, \\ 0, & \text{otherwise.} \end{cases}$$