University of Illinois **Spring 2025**

ECE 313: Problem Set 7

Due: Friday, March 28 at 7:00:00 p.m.
Reading: *ECE 313 Course Notes,* Section 3.6

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Scaling a uniform distribution]**
   Assume that you have a random number generator that generates a uniformly distributed random variable X over the interval $[-3, 5]$. Find a linear function g such that $g(X) = Y$, where Y is uniformly distributed over $[0, 1]$.

2. **[Scaling PDFs]**
   Suppose that X and Y are the sampled values of two different audio signals. An audio signal is said to be "spiky" if $P\{|Z| > 3\sigma_Z\} > 0.01$, i.e., one-in-hundred samples has a large amplitude.

   (a) Suppose that X is a uniformly distributed random variable, scaled so that it has zero mean and unit variance. What is $P\{|X| > 3\sigma_X\}$? Be sure to consider both positive and negative values of X.

   (b) Suppose that Y is a Laplacian random variable with a pdf given by:

   $$f_Y(u) = \frac{\lambda}{2} e^{-\lambda|u-\mu|}, \qquad -\infty < u < \infty$$

   where λ and μ are chosen so that $E[Y] = 0$ and and $\mathrm{Var}(Y) = 1$. What is $P\{|Y| > 3\sigma_Y\}$? Be sure to consider both positive and negative values of Y.

3. **[Gaussian Random Variables]**
   The random variable X has a $\mathcal{N}(-3, 16)$ distribution. Determine the following probabilities using the normal tables in Section 6.4 of the class notes. If a value is not in the table, round and use the closest value.

(a) $P(X = 0)$

(b) $P(|X + 3| \geq 4)$

(c) $P(X^2 < 4)$

4. **[Communication in Gaussian Noise]**
A wireless communication system consists of a transmitter and a receiver. The transmitter sends a signal x, and the receiver observes

$$Y = x + Z,$$

where Z is an noise term, modeled as a Gaussian random variable, independent from x, with mean $\mu_Z = 0$ and variance $\sigma_Z^2 = 1$.

(a) Suppose the transmitted signal is $x = 1$. What is the pdf of the received signal Y?

(b) Now suppose the transmitted signal can be either $x = -1$ or $x = 1$. The receiver uses the following decoding rule: if $Y > 0$, it declares that $x = 1$; if $Y \leq 0$, it declares that $x = -1$. Assuming that the transmitter sends -1 or $+1$ with probability $1/2$ each, what is the receiver's error probability?

5. **[Gaussian Approximation]**
You go to a casino and decide to play a game in which, with probability 0.4, you win 1 dollar, and with probability 0.6, you lose 1 dollar. You decide to play this same game repeatedly 100 times. Let $X_i \in \{-1, 1\}$ represent your earnings from the ith game, for $i = 1, \ldots, 100$. Assume that $X_1, X_2, ..., X_{100}$ are all independent. Let $X = \sum_{i=1}^{n} X_i$ be your total earnings (which may be negative).

(a) Let $Z_i = (X_i + 1)/2$, for $i = 1, \ldots, 100$ and $Z = \sum_{i=1}^{100} Z_i$. Notice that Z_i is a binary indicator of whether the ith game was won. What is the distribution of Z?

(b) Express the event $\{X \geq 10\}$ in terms of Z, and use the Gaussian approximation with continuity correction to compute $P\{X \geq 10\}$.

(c) Express the event $\{X = 0\}$ in terms of Z and use the Gaussian approximation with continuity correction to compute $P\{X = 0\}$.