**University of Illinois** **Spring 2025**

# ECE 313: Problem Set 6

**Due:** Friday, March 14 at 7:00:00 p.m.
**Reading:** *ECE 313 Course Notes,* Sections 3.2 - 3.5

**Note on reading:** For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

**Note on turning in homework:** Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. [**Probability Density Functions**]
   The lifetime of a certain kind of radio tube is a random variable $X$ with pdf
   $$f(x) = \frac{C}{x^2}, \text{ for } x > 100,$$
   and zero otherwise.
   (a) Find the constant $C$.
   (b) Assuming that tubes break down independently, find the probability that at least 3 out of 5 tubes will break down within the first 150 hours of operation.
   (c) What can you say about the expectation and variance of $X$?

2. [**Exponential Random Variables**]
   The amount of time, in hours, of a computer working without breaking down is a random variable with pdf
   $$f(x) = \lambda \exp(-\frac{x}{100}), \text{ for } x > 0,$$
   and zero otherwise.
   (a) Find $\lambda$.
   (b) What is the probability that the computer will function between 50 and 150 hours before breaking down?

(c) What is the probability that the computer will function less than 100 hours?

3. **[Uniform Continuous Random Variables]**
   A stick of length 2 is split at a point $U$ that is uniformly distributed over $(0, 2)$.

   (a) Determine the expected length of the part of the stick that contains a given point $p$.

   (b) Which choice of $p$ maximizes the expected length of the piece described in the previous part?

4. **[Departure time]**
   Suppose that if you are $s$ minutes early for an appointment, you incur a cost of $cs$, $c > 0$, and if you are $s$ minutes late, you incur a cost $ks$, $k > 0$. Also, suppose that the travel time from your house to the location of your appointment is a continuous random variable $X$ with pdf $f(x)$ and CDF $F_X(x)$. Determine the time at which you should depart if you want to minimize your expected appointment cost. (Hint: You may find the Leibniz integral rule useful for minimizing your expected appointment cost).

5. **[Poisson Processes]**
   Let $\{N(t), t \in [0, \infty)\}$ be a Poisson Process with rate $\lambda = \frac{1}{2}$.

   (a) Find the probability of no arrivals in the intervals $[0, 1]$, $[1, 2]$ and $[2, 3]$. Compare these probabilities with each other and with the probability of no arrivals in $[0, 3]$.

   (b) Find the probability that there are exactly 1, 2, and 3 arrivals in the intervals $[0, 1]$, $[2, 4]$ and $[\frac{1}{2}, 3]$, respectively.

   (c) Let $X_1$ be the first arrival time in the Poisson Process. Show that, given $N_t = 1$, $X_1$ is uniformly distributed in $(0, t]$.