University of Illinois **Spring 2024**

ECE 313: Exam I Conflict

Tuesday, February 27, 2024
7:00 p.m. — 8:15 p.m.

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section: ☐ A, MWF at 10 am (Milenkovic) ☐ B, MWF at 11 am (Katselis) ☐ C, MWF at 1 pm (Shanbhag) ☐ D, MWF at 2pm (Bastopcu) ☐ CSP, Chicago (Shanbhag)

Instructions

This exam is closed book and closed notes except for one 8.5"×11" sheet of notes: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, microscopes, etc. are not allowed.

The exam consists of four problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. **Draw a small box around each of your final numerical answers.**

Grading

1. 20 points	________
2. 35 points	________
3. 30 points	________
4. 15 points	________
Total (100 points)	________

1. **[20 points]** Suppose we roll two fair dice. Let X, Y be the corresponding outcomes. Consider the events $A = \{X \in \{1, 2, 3\}\}$, $B = \{X \in \{3, 4, 5\}\}$, $C = \{X + Y = 9\}$.

   (a) [10 points] **True or False**: $P(ABC) = P(A)P(B)P(C)$. Justify your answer.

   (b) [10 points] Are A, B, C independent? Justify your answer.

2. **[35 points]** Suppose that the random variable X has the following pmf:

$$p_X(k) = \begin{cases} c_1 \left(\frac{1}{3}\right)^k, & \text{if } k = -2, 0, \\ c_2, & \text{if } k = m, m+2, \\ 0, & \text{otherwise.} \end{cases}$$

Note: If you cannot find a solution for some parts of the problem, use the symbols without knowing their numerical values to express the solutions for other parts.

(a) [10 points] Given that $p_X(m+2) = 5p_X(0)$, what are the values of the constants c_1 and c_2 so that we have a valid pmf?

(b) [10 points] Find the value of m such that $E[X] = 0$, using the constants c_1, c_2 found in part (a).

(c) [5 points] With the constants c_1, c_2, found in part (a), and the value of m found in part (b), find the variance of the random variable X, i.e., $Var(X)$.

(d) [10 points] Find $P(|X| \geq 2)$ and $E[|X|]$, for the values of the constants c_1, c_2 found in part (a) and the value of m found in part (b).

3. **[30 points]** The parts of this problem are independent.

(a) [10 points] Recall that for two events A, B, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. What is the probability that *exactly* one of these two events occurs? Justify your answer.

(b) [15 points] Suppose that a biased coin with $P(H) = \frac{1}{4}$ is repeatedly tossed. What is the probability that at the nth toss, the numbers of heads and tails are equal for an even n? What is the corresponding probability when n is odd? You can leave your answer as an unsimplified expression.

(c) [5 points] Suppose X, Y are two nonnegative random variables, both upper bounded by 10. Let $E[X] = E[Y] = 2$, $Var(X) = 1$ and $Var(Y) = 2$. Use Markov's inequality to upper bound $P(X^2 + Y^2 \leq 20)$.

4. **[15 points]** The parts of this problem are independent.

(a) [10 points] Suppose that X takes values in the set $\{2, 5, 8, ..., 3n + 2\}$, each with equal probability. Assume that $X = 17$ is observed. Find $\hat{n}_{ML}$.

(b) [5 points] Suppose that X is a geometric random variable with parameter p. Assume that instead of a numerical observation of X, i.e., an event of the form $\{X = k\}$, we are provided with an event of the form $\{X \in \{k_1, k_2\}\}$ as observation. In other words, we conducted our experiment and found that X could be either k_1 or k_2. Let $k_1 = 2$ and $k_2 = 3$. Propose a possible estimate $\hat{p}$ of p in this case.