University of Illinois Spring 2024

ECE 313: Exam II Conflict

Tuesday, April 9, 2024
7:00 p.m. — 8:15 p.m.

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Section: ☐ A, MWF at 10 am (Milenkovic) ☐ B, MWF at 11 am (Katselis)
☐ C, MWF at 1 pm (Shanbhag) ☐ D, MWF at 2pm (Bastopcu) ☐ CSP, Chicago (Shanbhag)

Instructions

WRITE YOUR NAME AND NETID ON TOP OF EVERY PAGE. This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of four problems worth a total of 100 points. The problems are not necessarily weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. You may use the back of the previous page as scratch paper, but we will only grade answers in the space provided. Draw a small box around each of your final numerical answers.

Grading	
1. 25 points	_____
2. 25 points	_____
3. 25 points	_____
4. 25 points	_____
Total (100 points)	_____

1. **[25 points]** A software development company claims that 20% of their software applications have critical bugs. However, a group of independent software testers believes that the actual proportion of buggy software is equal to 40%. So, software testers independently sampled 3 software applications from the company's portfolio and noted the number of buggy software.

   (a) (8 points) Write out the likelihood matrix (table) for this problem. Find the ML decision rule using the likelihood matrix (table).

   (b) (5 points) Find $p_{false\,alarm}$, p_{miss}, and p_e for the ML rule. To calculate p_e, assume that the prior probability of the company's claim being true is equal to 1/3.

(c) (7 points) Find the MAP rule, assuming that the prior probability of the company's claim being true is equal to 1/3.

(d) (5 points) Find $p_{false\,alarm}$, p_{miss}, and p_e for the MAP rule.

2. **[25 points]** A Geiger counter records the arrival of subatomic particles governed by a Poisson process of rate equal to $\lambda = 3$ particles/second.

   (a) (5 points) Find the probability that you see 10 particles arriving in the first minute.

   (b) (10 points) Find the probability that the Geiger counter did not see any arrivals within the first 10 seconds, the time interval $[10, 30]$ seconds as well as in the time interval $[30, 40]$ seconds.

(c) (10 points) What is the probability that the Geiger counter recorded 1 arrival in each of the three time intervals $[0, 3]$, $[1, 4]$, and $[2, 5]$ seconds?

3. **[25 points]** Suppose a *fair* die is flipped 162 times. Let the event A be defined as:

$A = \{(\text{number of times a number} \leq 4 \text{ shows}) \leq (\text{number of times a number} > 4 \text{ shows}) + 4\}$.

(a) (4 points) Let X be the random variable denoting a number ≤ 4 shows. Express A in terms of X. What is the probability mass function of X?

(b) (5 points) Write down an expression for the exact value of $P(A)$.

(c) (8 points) Using the Gaussian approximation with the continuity correction, approximate the value of $P(A)$ via the Q function. Leave the expression in terms of the Q function.

(d) (8 points) Let B denote the event $\{X = 55\}$. Using the Gaussian approximation with the continuity correction, approximate the value of $P(B)$ via the Φ function. Leave the expression in terms of the Φ function.

4. **[25 points]** Let X be a random variable with pdf

$$f_X(x) = cx(1-x), \quad a < x < b,$$

and $f_X(x) = 0$, otherwise. Here, $c > 0$.

(a) (5 points) Find the possible values of a and b.

(b) (10 points) Find c in terms of a and b.

(c) (10 points) Suppose that $a < \frac{1}{2} < b$. Find $P(X > \frac{1}{2})$ in terms of a and b.