University of Illinois **Spring 2024**

ECE 313: Problem Set 10

Due: Friday, April 5 at 7 p.m.

Reading: *ECE 313 Course Notes,* Sections 3.7, 3.8.1, 3.8.2, and 3.10

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Maximum Likelihood Parameter Estimation]**
   Random variable $Y = aX + 1$. Answer the following:

   (a) If $X \sim \text{Unif}[0,1]$ and $Y = 3$ is observed, find the maximum likelihood estimate $\hat{a}_{\text{ML}}$ and the density function $f_Y(v)$.

   (b) If $Y \sim \text{Unif}[0,1]$ and $X = 3$ is observed, find the maximum likelihood estimate $\hat{a}_{\text{ML}}$ and the density function $f_X(u)$.

2. **[Function of a RV]**
   Let $X \sim [0, \pi]$ be a uniformly distributed random variable and let $Y = g(X) = a\cos X$, $a > 0$. Find the pdf $f_Y(v)$ of Y.

3. **[Function of a RV II]**
   Suppose that X is uniformly distributed over the interval $[-5, 5]$, and the function $h(u)$ is defined as:

$$h(u) = \begin{cases} -\frac{u}{2} & \text{if } -5 \le u < 0, \\ 2u & \text{if } 0 \le u < 2, \\ u^2 & \text{if } 2 \le u \le 5. \end{cases} \tag{1}$$

   We define the random variable Y as $Y = h(X)$.

   (a) Sketch the pdf of X and sketch $h(u)$. Identify the support of Y. Determine whether Y is a continuous-type or discrete-type random variable.

(b) Find $F_Y(u)$, the CDF of Y.

(c) Find the distribution of Y, i.e., the pdf (if Y is a continuous-type r.v.) or the pmf (if Y is a discrete-type r.v.). Find $E[Y]$, the expected value of Y.

4. **[Function of a RV III]**
Suppose that X is uniformly distributed over the interval $[0, 10]$, and the function $h(u) = \lceil u \rceil \bmod(7)$ for $0 \leq u \leq 10$. We define the random variable Z as $Z = h(X)$.

(a) Sketch the pdf of X and sketch $h(u)$. Identify the support of Z. Determine whether Z is a continuous-type or discrete-type random variable.

(b) Find the distribution of Z, i.e., the pdf (if Z is a continuous-type r.v.) or the pmf (if Z is a discrete-type r.v.).