ECE 313: Problem Set 8

Due: Friday, March 22 at 7 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.3 - 3.5

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Customer support center]**
   Suppose the number of calls into a customer support center in any time interval is a Poisson random variable with mean 5 calls per minute.
   (a) What is the probability that there will be four calls in an interval of 2 minutes?
   (b) What is the probability that there are at least three calls in an interval of one minute?

2. **[Poisson Process Intervals]**
   Let N_t be a Poisson process with rate $\lambda > 0$. Your answers may include λ for the following questions.
   (a) Find $P(N_5 = 7)$.
   (b) Find $P(N_8 - N_3 = 7)$ and $E[N_8 - N_3]$.
   (c) Find $P(N_8 - N_3 = 7 | N_5 - N_4 = 5)$.
   (d) Find $P(N_5 - N_4 = 5 | N_8 - N_3 = 7)$.

3. **[Uniform Distributions]**
   Let U and V be independent random variables, both uniformly distributed on $[0, 1]$. Find the probability that the quadratic equation $x^2 + 2Ux + V = 0$ has two real solutions.

4. **[Exponential Distributions I]**
   Let G be an exponentially distributed random variable with $\lambda = \ln(3) + 1$.

(a) Find a simple expression for $P(G \geq g)$.

(b) Find $P(G > 4|G > 2)$, $P(G < 4|G > 2)$ and $P(G > 2|G < 4)$.

5. **[Understanding the Exponential]**
   Suppose X has the exponential distribution with parameter $\lambda > 0$.

   (a) Find λ such that $E[X^4] - E[X^2] = 0$.

   (b) Express $P(\lceil X^3 \rceil = 8) + P(e < e^X < e^2)$ in terms of λ.

6. **[(Extra Credit) Exponential Distributions II]**
   This is a problem for extra credit, which requires a little bit more thought.

   (a) Find the CDF of the minimum of two independent exponential random variables with parameter λ. Hint: work with $1 - F_X(x)$, where $F_X(x)$ is the CDF of the minimum. Use the independence property.

   (b) You have a digital device that requires two batteries to operate. To be on the safe side, you buy 3 types of batteries (say, marked as $1, 2, 3$), each of which has a lifetime that is exponentially distributed with parameter λ and operates or fails independently of all the other batteries. Initially you install two batteries, say 1 and 2. When one of these two batteries fails, you replace it with battery 3. What is the expected total time until your device stops working?

   (c) In the scenario of part (b), what is the probability that battery 1 is the last battery that still works?