University of Illinois **Spring 2024**

ECE 313: Problem Set 7

Due: Friday, March 8 at 7 p.m.
Reading: *ECE 313 Course Notes,* Sections 2.12 - 3.2

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. [**Union bound**]
   Suppose the probability that a professor is on time when teaching their lab on any given day is at least 0.99.

   (a) Bound the probability that the professor is late at least once over 30 labs. Bound the probability that the professor is late at least once over 200 labs. Do not make any independence assumptions.

   (b) Now, assume that two professors jointly teach a lab, and that both have to be present to run the lab. The professors are late independently of each other with the probabilities stated at the beginning. Bound the probability that at least one lab will not start on time over a span of 10 labs.

2. [**CDFs**]
   You choose a real number uniformly at random in the interval $[a, b]$, multiply it by 2, and call it X. By uniformly at random, we mean all intervals in $[a, b]$ that have the same length must have the same probability.

   (a) Find the CDF of X.

   (b) Find the PDF of X.

   (c) Find the expected value of X and X^3.

3. **[More on CDFs]**
Let X be a continuous random variable with the following pdf

$$f_X(x) = \begin{cases} c^2\, e^{-5x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

   (a) Find c.

   (b) Find the CDF of X, $F_X(x)$, using the value of c you found in Part (a).

   (c) Find $P(1 < X < 3)$, using the value of c you found in Part (a).

4. **[Optimal Location of a Fire Station]**
A fire station needs to be located on a road that is L miles long lined with houses with the first house at the origin. The location of house fires is assumed to be uniformly distributed in the interval $[0, L]$.

   (a) Determine the optimum location of the fire station such that the average distance from the fire station to the fire location is minimized.

   (b) Suppose, to put out a fire, it costs the city an amount in dollars equal to the square of the distance of the fire station from the location of the fire. Find the average cost of locating the fire station at the optimum distance obtained in Part (a).