University of Illinois **Spring 2024**

ECE 313: Problem Set 4

Due: Friday, February 16 at 7 p.m.
Reading: *ECE 313 Course Notes,* Sections 2.4-2.7.

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Binomial and Poisson]**
   Solve the following questions.
   (a) Let $X \sim \text{Bin}(n, p)$. Find $P(X \text{ is odd})$ in terms of n and p.
   (b) Let $Y \sim \text{Poi}(\lambda)$. Find $P(Y \text{ is odd})$ in terms of λ.
   (c) Suppose that $n \to \infty$ and $p \to 0$ such that $np = \lambda$. Verify that your answer in part (a) converges to the answer in part (b).

2. **[Waiting for Success]**
   Consider a sequence of independent $\text{Ber}(p)$ trials until the first success. Let X be the corresponding number of trials.
   (a) Compute $E[X|X > 10]$ for $p = \frac{1}{4}$.
   (b) Compute $E[\sin(\frac{[2X+1]\pi}{2})]$ for $p = \frac{1}{4}$.

3. **[Bins and Balls]**
   A total of n balls are chosen sequentially and at random without replacement from a bin containing r red and b blue balls, where $r + b \geq n$.
   (a) What is the probability that for $n = 3$, the last ball drawn is red given that the first two were also red?
   (b) For a general n, given that k out of the n balls are blue, what is the conditional probability that the first ball chosen is blue?

4. **[More on Throwing Dice]**
Two fair dice are thrown. Let E denote the event that the sum of the dice is 7. Let F denote the event that the first die equals 4 and let G be the event that the second die equals 3.

   (a) Are E and F independent events? Are E and G independent events?

   (b) Are E and $F \cap G$ independent events?

   (c) Are the events E, F and G mutually independent events?

   (d) Provide an example of two events that are both mutually exclusive and independent (your example should pertain to the dice problem stated).

5. **[Bumped From The Flight]**
Suppose that 105 passengers hold reservations for a 100-passenger flight. The number of passengers who show up at the gate can be modeled as a binomial random variable $X \sim Binomial(105, 0.9)$.

   (a) On average, how many passengers show up at the gate?

   (b) If $X \leq 100$, everyone who shows up gets to board. Find $P(X \leq 100)$.

   (c) Explain why the number of no-shows can be modeled as a binomial random variable $Y \sim Binomial(105, 0.1)$.

   (d) Notice that the probability that everyone who shows up gets to board can also be expressed as $P(Y \geq 5)$. Use the Poisson approximation to compute $P(Y \geq 5)$ and compare your answer to the exact answer that you found in part (b).