University of Illinois **Spring 2024**

ECE 313: Problem Set 3

Due: Friday, February 9 at 7 p.m.
Reading: *ECE 313 Course Notes,* Sections 2.4 - 2.7.

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Conditional Probability]**
   Suppose Z_1, Z_2, Z_3 are i.i.d. Bernoulli random variables with parameter $p = \frac{1}{3}$. The random variable $S = Z_2 + Z_3$ if $Z_1 = 1$, and $S = Z_2 - Z_3$ if $Z_1 = 0$.

   (a) Find $P(S = 1)$.

   (b) Find $P(Z_1 = 1|S = 1)$.

   (c) Find $P(Z_2 = 1|S = 0)$.

2. **[Career Fair]**
   A graduating ECE student goes to career fair booths in the technology sector. His/her likelihood of receiving an off-campus interview invitation after a career fair booth visit depends on how well he/she did in ECE 313. Specifically, an A in 313 results in a probability $p = 0.9$ of obtaining an invitation, whereas a C in ECE 313 results in a probability $p = 0.2$ of receiving an invitation.

   (a) On average, how many booth visits must an A student make before getting an off-campus interview invitation? How about a C student?

   (b) Each student is allowed to visit exactly 5 booths during this career fair. What is the probability that the A student receives exactly two interviews? How about the C student?

   (c) Assume that each student visits 5 booths during a typical career fair. Find the probability that an A student in 313 will not get an off-campus interview invitation. Similarly, find the probability that a C student in 313 will get an invitation.

(d) Assume an A student already visited 10 booths without getting an interview, and a C student already visited 5 booths and received 2 interviews. If they both go visit 5 additional booths, who has a higher probability of landing an interview?

3. **[Customer support center]**
Suppose the number of calls into a customer support center in any time interval is a Poisson random variable with mean 4 calls per minute.

   (a) What is the probability that there will be two calls in an interval of 3 minutes?

   (b) What is the probability that there are at least three calls in an interval of one minute?

4. **[Probability theory and dieting]**
You were told that you need to go on a diet but you crave food since you have challenging probability problems to solve. You therefore decide to gamble to determine if you should eat a slice of pizza or not. You toss an *unfair* coin (you really want the pizza!) to decide if you should eat it. The bias is such that head shows up with twice the probability of tail, and you eat the slice if the toss is head. If the toss is tail, you continue (you really, really want that slice of pizza!), and toss the coin two more times. If the subsequent two tosses are both heads, then you finish off the pizza. If not, you decide it is not meant to be and give the pizza slice to your roommate (or dog – cats are too smart to eat pizza). *All tosses are performed mutually independently.*

   (a) Find the probability that you get to eat your slice of pizza.

   (b) What is the probability that the first toss was heads, given that you ate the slice of pizza?

5. **[Card problems]**
Suppose that you are dealt 3 cards in a sequential order, without replacement, from a standard deck of 52 cards. The draws are performed independently of each other.

   (a) What is the probability that you get exactly one heart?

   (b) Given that you got exactly one heart, what is the probability that the heart was on the first card you drew?

   (c) Are the two events of getting "exactly three 2s" and "at least two 2s" when dealt the three cards from the deck statistically independent?