## ECE 313: Problem Set 2

Due: Friday, February 2 at 07:00:00 p.m.
Reading: ECE 313 Course Notes, Sections 2.1-2.3
Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.
Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. Please write down your work and derivations. An answer without justification as of how it is found will not be accepted. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET \#
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. 5 points will be deducted for incorrectly assigned page numbers.

1. [Two more poker hands]

Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3 from the notes, with all possibilities being equally likely.
(a) $F L U S H$ is the event where all five cards have the same suit. Find $P(F L U S H)$.
(b) FOUR OF A KIND is the event that four of the five cards have the same number. Find P(FOUR OF A KIND).
2. [Principles of Counting]

A restaurant offers 6 entrees: $\left\{E_{1}, E_{2}, E_{3}, E_{4}, E_{5}, E_{6}\right\}$ and 8 sides: $\left\{S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}, S_{7}, S_{8}\right\}$. Alice and Bob want to choose 3 different entrees and 4 different sides as their dinner.
(a) How many different dinners consisting of 3 different entrees and 4 different sides are possible?
(b) Suppose that sides $\left\{S_{1}, S_{2}, S_{3}, S_{4}\right\}$ can only be ordered if $E_{1}$ is chosen as one of their entrees, and sides $\left\{S_{5}, S_{6}, S_{7}, S_{8}\right\}$ can be only ordered if both $E_{2}$ and $E_{3}$ are ordered as two of their entrees. With these constraints, how many different dinners consisting of 3 different entrees and 4 different sides are possible?

## 3. [A Karnaugh map for three events]

Let an experiment consist of rolling two fair 4 -sided dice, and define the following three events about the numbers showing:
$A=$ "the sum of the two numbers is even"
$B=$ "the sum of the two numbers is a multiple of 5 "
$C=$ "the number showing on the first die is (strictly) less than the number showing on the second die"
(a) Display the outcomes in a three-event Karnaugh map.
(b) Find $P((A \cup B) C)$.

## 4. [PMF, Mean and standard deviation]

Suppose two fair dice are rolled independently, so the sample space is $\Omega=\{(i, j): 1 \leq i \leq$ 6 , and $1 \leq j \leq 6\}$, and all outcomes are equally likely. Let $X$ be the random variable defined by $X(i, j)=\min \{i, j\}$, and let $Y$ be the random variable defined by $Y(i, j)=\max \{i, j\}$.
(a) Derive the pmf of $X$ and sketch it.
(b) Find the mean, $E[X]$, and standard deviation, $\sigma_{X}$, of $X$.
(c) Derive the pmf of $Y$ and sketch it.
(d) Find the mean, $E[Y]$, and standard deviation, $\sigma_{Y}$, of $Y$.

## 5. [PMF, Mean II]

Suppose that you (as a UIUC student) are planning on going to a concert in the Krannert Center. Ten of your friends are also interested in the same concert. Four of your friends are UIUC students, whereas the remaining six friends are not students. You randomly choose four of your friends to go with you. When you get to the Krannert Center, it turns out that UIUC students pay $\$ 10$ and those who are not students pay $\$ 50$ for their tickets. Let $X$ be the total amount of money you and your friends end up paying to get into the concert.
(a) What values can $X$ take?
(b) Find the pmf of $X$.
(c) Find the expected value of $X$.

## 6. [Gambling with Dice]

You roll a fair die. If you roll an even number $i$, you win $i$ dollars. If you roll an odd number, you lose $m$ dollars. Let $X$ denote the amount of money you win (a negative amount indicates a loss).
(a) Find the pmf of the random variable $X$. Note that the pmf will depend on $m$.
(b) If $E[X]=0$, find $m$.
(c) Fixing the value of $m$ to the value you found in part (b), find $\operatorname{Var}(X)$.

