ECE 313: Lecture 8
Independent events and independent random variables
Mean, variance, LOTUS (revisit)

Example: \( n \) pairs of shoes, each \( \in \{ L, R \} \)

Pick 2 shoes at random

1. What is the prob getting a pair \( (i, j) \) for some \( 1 \leq i < j \leq n \)?

2. """ "" "" getting a \( (i_L, i_R) \) for \( i \neq j \)

Solution (1) Total \# of possible sets of 2 shoes

\[
|\Omega| = \binom{2n}{2} = \frac{2n(2n-1)}{2}
\]

\( M = \{ (1_L, 1_R), (2_L, 2_R), \ldots, (n_L, n_R) \} \)

\[
\frac{|M|}{|\Omega|} = \frac{n \cdot 2}{2n(2n-1)} = \frac{1}{2n-1}
\]
Alternatively: $E_1 = "$ pick the first shoe $"

$E_2 = "$ pick the second shoe that matches w. the first one $"

We need:

$P(E_1, E_2) = P(E_1) P(E_2 | E_1) = 1 \cdot \frac{1}{2n-1} = \frac{1}{2n-1}$

$P(\text{one } 1, \text{ one } R^2) = \frac{n^2}{\binom{2n}{2}} = \frac{2n^2}{2n(2n-1)}$

$= \frac{n}{2n-1}$

Alternatively $= \frac{n}{2n-1}$
**Ex. 2:** Throw 2 dice. 

\[ S = \{ (i, j) : 1 \leq i, j \leq 6 \} \]

- **A:** "First dice is even"  
- **B:** "Sum of 2 dice is odd"  
- **C:** "Second dice is odd"

In this case:

\[ P(A) = \frac{18}{36} = \frac{1}{2} \]

\[ P(C) = \frac{18}{36} = \frac{1}{2} \]

\[ P(A \cap C) = \frac{9}{36} = \frac{1}{4} \]

\[ \frac{P(AC)}{P(C)} = P(A) \]

\[ \frac{P(AC)}{P(C)} = P(A) \]
Definition: Two events $A$ and $B$ are **mutually independent** if

$$P(AB) = P(A)P(B)$$

**Exercise:** Is $A$ and $B$ independent in the previous example?
\( x \) is a discrete random variable

\[
P \left( \{ x = k \} \right) = f_x(k)
\]

\[
E[x] = \sum_k k \cdot f_x(k)
\]

\[
E[x^2] = \sum_k k^2 \cdot f_x(k)
\]

\[
\sigma_x^2 = \text{Var}[x] = E[x^2] - (E[x])^2
\]

\[\text{LOTUS}\]

\[
E[g(x)] = \sum_k g(k) \cdot f_x(k)
\]