ECE 313: Lecture 27
Joint CDFs (Ch 4.1)
Joint pmfs (Ch 4.2)

**Example:** \( T \sim \text{Exp} \left( \lambda \right) \)

\[
\begin{align*}
    f_T(u) &= \lambda e^{-\lambda u} & u \geq 0 \\
    F_T(u) &= 1 - e^{-\lambda u} \\
    h_T(u) &= \lambda
\end{align*}
\]

**Review:** Failure Rate

**Serial**

\[
\begin{array}{c}
    \text{src} \quad T_1 \quad T_2 \quad \text{ter} \\
    \text{max}
\end{array}
\]

Life time of whole network

\[
T = \min \{ T_1, T_2 \}
\]

\[
1 - F_T(t) = \mathbb{P} \left\{ T > t \right\} = \mathbb{P} \left\{ T_1 > t, T_2 > t \right\}
\]

If \( T_1, T_2 \) are ind.

\[
= \mathbb{P} \left\{ T_1 > t \right\} \cdot \mathbb{P} \left\{ T_2 > t \right\} = 1 - F_T(t)
\]

**Parallel**

\[
\begin{array}{c}
    \text{src} \quad 1 \quad 2 \quad \text{ter} \\
    \text{min}
\end{array}
\]

Life time of whole network

\[
T = \max \{ T_1, T_2 \}
\]

\[
\begin{align*}
    \mathbb{P} \left\{ T > t \right\} &= \mathbb{P} \left\{ T_1 > t \right\} \cup \left( \mathbb{P} \left\{ T_2 > t \right\} \right) \\
    &= \mathbb{P} \left\{ T_1 > t \right\} + \mathbb{P} \left\{ T_2 > t \right\} - \mathbb{P} \left\{ T_1 > t, T_2 > t \right\}
\end{align*}
\]
Joint CDF

Before: often assume $X$ and $Y$ are independent

Now, more general $X$ and $Y$ are dependent

Most important: $Y = \text{Predictor}_Y(X)$

Model $X$ and $Y$ jointly

$$(X, Y) \sim F_{X, Y} \text{ or } f_{X, Y}$$

Ex: $\begin{array}{c|cc} & Y=1 & Y=0 \\ \hline X=0 & 0.1 & 0.3 \\ X=1 & 0.3 & 0.7 \end{array}$

$p_{X,Y}(u, v) = p_{X,Y}(X=0, Y=0) \cdot 

\cdot f_{X,Y}(u, v) \\

u, v \in \mathbb{R}, 0, 1 \}$

$\mathbb{P} \{X = 0\} = \mathbb{P} \{X = 0, Y = 0\} + \mathbb{P} \{X = 0, Y = 1\}$
1. [8+6+10+6 points] The joint pmf of two discrete-type random variables is as shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th>X = 1</th>
<th>X = 2</th>
<th>X = 3</th>
<th>X = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y = 1</td>
<td>0.2</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Y = 2</td>
<td>0</td>
<td>0.25</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>Y = 3</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Find the marginal pmfs of X and Y.

\[ p_X(u) = \sum_y p_{X,Y}(u, v) \]
\[ p_Y(v) = \sum_u p_{X,Y}(u, v) \]

(b) Are X and Y independent? Justify your answer.

\[ p_{X,Y}(u, v) = p_X(u) p_Y(v) \quad \text{for all } u, v \]
\[ N_o: \quad 0 = p_{X,Y}(2, 1) \neq p_X(2) p_Y(1) \]

(c) Find the conditional pmf \( p_{Y|X}(v|4) \).

\[ p_{Y|X}(v|4) = \frac{p_{X,Y}(u, v)}{p_X(u)} \]

(d) Find \( P\{X < Y\} \).

\[ 0.05 \]