University of Illinois Spring 2019

ECE 313: Hour Exam II

Wednesday, April 10, 2019
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ____________________________________

NetID: ____________________________________

Signature: ____________________________________

Section:
☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 12 points	________
2. 28 points	________
3. 22 points	________
4. 22 points	________
5. 16 points	________
Total (100 points)	________

1. **[6+6 points]** Suppose X is uniformly distributed on $[0, 2]$.

   (a) Find $E(X^2)$.

   (b) Find $F_X(1)$.

2. **[10+10+8 points]** Three random variables are defined: X is exponentially distributed with parameter $\lambda = 2$, Y is Gaussian with $N(2, 9)$ and U is uniformly distributed in $[0, 1]$. Answer the following:

   (a) Find a function g such that $g(X) = U$. Assume that g is non-decreasing and g^{-1} exists. Show steps for full credit.

   (b) Find h such that $h(U) = Y$. Assume that h is non-decreasing and h^{-1} exists. Show steps for full credit.

(c) Find a function k such that $k(X) = Y$. Assume that k is non-decreasing and k^{-1} exists. Show steps for full credit.

3. **[12+10 points]** Consider the binary hypothesis problem in which the pdfs of X under hypotheses H_0 and H_1 are given by

$$f_0(u) = \begin{cases} \frac{1}{2} & \text{if} \quad 0 \le u \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$f_1(u) = \begin{cases} u & \text{if} \quad 0 \le u < 1 \\ 2-u & \text{if} \quad 1 \le u \le 2 \\ 0 & \text{otherwise} \end{cases}$$

with priors $\pi_1 = 2\pi_0$.

(a) Write an expression for the likelihood function $\Lambda(u)$ and find the MAP rule.

(b) Calculate p_{miss}, $p_{\text{false-alarm}}$, and error probability p_e.

4. **[8+6+8 points]** Suppose X and Y are discrete-type random variables with the joint pmf given by
$p_{X,Y}(u,v) = \frac{u+v}{32}, u = 1, 2, v = 1, 2, 3, 4.$

(a) Find $p_X(u)$, the marginal pmf of X.

(b) Find $P\{Y = 2X\}$

(c) Find $p_{X|Y}(u|2)$, the conditional probability of X given $Y = 2$.

5. **[6+10 points]** Suppose X and Y are independent random variables with probability density functions $f_X(u) = \begin{cases} 3u^2 & 0 \le u \le 1 \\ 0 & \text{else,} \end{cases}$ and $f_Y(v) = \begin{cases} 2v & 0 \le v \le 1 \\ 0 & \text{else,} \end{cases}$ respectively.

(a) Find the joint pdf of (X, Y).

(b) Find $P\{X < Y\}$.