University of Illinois Spring 2019

ECE 313: Hour Exam II

Wednesday, April 10, 2019
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section:
☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 26 points ____________
2. 12 points ____________
3. 20 points ____________
4. 22 points ____________
5. 20 points ____________

Total (100 points) ____________

1. **[5+5+11+5 points]** Consider a Poisson process with rate 1. Define X to be the total number of counts during $[0, 3]$, and Y to be the number of counts during $[0, 1]$. Let T be the time of the first count. Express your answers to the following questions in terms of e.

   (a) Find $P(X = 2)$.

   (b) Find $P(T > 2)$.

(c) Find $P(X = 2|Y = 1)$.

(d) Find $E(T|T > 2)$

2. **[12 points]** Let $T_1, T_2, \cdots, T_{10}$ be i.i.d. exponentially distributed random variables with parameter λ. Suppose we observe that five out of the ten random variables have values greater than 1. Find the ML estimate, $\hat{\lambda}$, of λ.

3. **[8+12 points]** A random variable X has a $N(2, 9)$ distribution and $Y = 3X + 2$.

(a) Find an expression for the density function of Y, $E[Y]$ and $\text{Var}(Y)$.

(b) Find $P\{|3X + 1| > 2\}$ in terms of the CDF of a standard normal ($\Phi(x)$.)

4. **[12+10 points]** Consider the binary hypothesis problem in which the pdfs of X under hypotheses H_0 and H_1 are given by

$$f_0(u) = \begin{cases} \frac{1}{2} & \text{if} \quad 0 \leq u \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$f_1(u) = \begin{cases} u & \text{if} \quad 0 \leq u < 1 \\ 2-u & \text{if} \quad 1 \leq u \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

with priors $\pi_1 = 2\pi_0$.

(a) Write an expression for the likelihood function $\Lambda(u)$ and find the MAP rule.

(b) Calculate p_{miss}, $p_{\text{false-alarm}}$, and error probability p_e.

5. **[8+8+4 points]** Let X and Y be continuous-type random variables with joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 2, & 0 \le v \le u \le 1 \\ 0, & \text{else.} \end{cases}$$

Then the marginal pdf of Y is $f_Y(v_o) = \begin{cases} 2(1-v_o), & 0 \le v_o < 1 \\ 0, & \text{else.} \end{cases}$

(a) Find $f_X(u)$, the marginal pdf of X.

(b) Find $f_{X|Y}(u|v_o)$, the conditional pdf of X given Y.

(c) Are X and Y independent? Why?