University of Illinois Spring 2019

ECE 313: Hour Exam I

Wednesday, February 27, 2019
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ______________________________

NetID: ______________________________

Signature: ______________________________

Section:
☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5”×11” sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 20 points ____________
2. 24 points ____________
3. 22 points ____________
4. 16 points ____________
5. 18 points ____________

Total (100 points) ____________

1. **[6+14 points]** An experiment consists of observing the content of an 8-bit register. We assume that all 256 byte values are equally likely to be observed.

   (a) Let A denote the event that the least significant bit (LSB) is a ZERO. What is $P(A)$?

   (b) Let B denote the event that the register contains 5 ONEs and 3 ZEROes. What is $P(B|A)$?

2. **[10+6+8 points]** A 3-faced die is rolled twice. $\omega_1 \in \{1, 2, 3\}$ and $\omega_2 \in \{1, 2, 3\}$ are the outcomes of each roll. The random variable X takes values in the set $\{2\omega_1 + \omega_2\}$.

(a) Determine the set of values u_i that X can take and the probability mass function $P_X(u_i)$.

(b) Calculate $E[X]$.

(c) Calculate $E[\frac{1}{2X}]$.

3. [**8+8+6 points**] Bob plays a lottery game. He spends \$2 each week to purchase a lottery ticket for this game. Suppose the probability of winning a \$4 prize on each ticket is 1/30, and the probability of winning the Jackpot (the grand prize) on each ticket is 1/300,000,000.

   (a) Bob will purchase a total of 52 tickets in the year 2019 (=52 weeks). What is the expected number of times that Bob wins a \$4 prize this year? Present your answer in an irreducible fraction.

   (b) Bob has purchased 8 tickets since Jan 1, 2019. What is the probability that Bob has won the \$4 prize twice since Jan 1, 2019? Write down the expression for your answer without calculating the final numerical value.

(c) Find the expected number of tickets needed until a Jackpot is won by Bob (suppose he could play the game forever).

4. **[8+8 points]** Suppose a random variable X is uniformly distributed on $\{2, 4, 6, 8, 10, ..., 2n\}$.

(a) We observe $X = 10$. Find the ML estimate of n.

(b) Suppose we make two independent observations of X. Denote the observations by X_1 and X_2. We have $X_1 = 20$ and $X_2 = 16$. Find the ML estimate of n.

5. **[12+6 points]** Suppose Z_1, Z_2, Z_3 are i.i.d. Bernoulli random variables with parameter $p = \frac{1}{2}$. The random variable $S = Z_2 Z_3$ if $Z_1 = 1$, and $S = Z_2 + Z_3$ if $Z_1 = 0$.

   (a) Find $P(S = 1)$.

   (b) Find $P(Z_1 = 1 | S = 1)$.