**University of Illinois** Spring 2018

# ECE 313: Hour Exam II

Wednesday, April 11, 2018
8:45 p.m. — 10:00 p.m.

**Name: (in BLOCK CAPITALS)** _____

**NetID:** _____

**Signature:** _____

**Section:**
☐ C, 10:00 a.m. ☐ G, 11:00 a.m. (Alvarez) ☐ D, 11:00 a.m. (Dominguez-Garcia)
☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

## Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided. DO NOT convert answers to decimal fractions (for example, write $\frac{3}{4}$ instead of 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

| Grading | |
|---|---|
| 1. 22 points | _____ |
| 2. 22 points | _____ |
| 3. 18 points | _____ |
| 4. 18 points | _____ |
| 5. 20 points | _____ |
| Total (100 points) | _____ |

1. **[22 points]** A shuttle bus arrives at a bus stop at 12:00PM and leaves at 12:02PM. People arrive at the bus stop to catch the shuttle according to a Poisson process with rate $\lambda = 0.1$ person per minute. Assuming there are no people waiting for the shuttle bus upon its arrival at 12:00PM, find the probability of the following events. (Leave your answers in powers of e, e.g. $ae^b$ for some constants $a$ and $b$.)

   (a) $E_1$: The shuttle does not pick any new passenger at the bus stop.

   (b) $E_2$: The shuttle picks up two new passengers at the bus stop.

(c) $E_3$: The only passenger the shuttle picks arrives at the bus stop after 12:01PM.

2. **[22 points]** Suppose $X$ and $Y$ are independent random variables such that $X$ is uniformly distributed over the interval $[0, 1]$ and $Y$ is exponentially distributed with parameter $\lambda > 0$.

   (a) Find the joint CDF $F_{X,Y}$.

   (b) Find $P(Y = X)$.

(c) Find $P(Y \leq 4X)$.

3. **[18 points]** Consider a binary hypothesis testing problem where

$$H_0: \quad f_0(y) = \begin{cases} y+1, & y \in (-1,0) \\ -y+1, & y \in (0,1) \\ 0, & \text{else.} \end{cases}$$

$$H_1: \qquad f_1(y) = \begin{cases} \frac{1}{3}, & y \in (0,3) \\ 0, & \text{else.} \end{cases},$$

(a) Determine the ML decision rule.

(b) Forget about the result in part $(a)$. Determine a decision rule that yields $p_{miss} = \frac{5}{6}$.

4. **[18 points]** The two parts of the problem are unrelated.

   (a) The lifetimes of light bulbs (in years) produced by two companies, *Fos* and *Illuminati*, follow exponential distributions with parameters $\lambda = 1$ and $\lambda = 2$, respectively. You purchased a random lightbulb from a store that does not carry manufacturer labels, but carries the same number of products by *Fos* and *Illuminati*. What is the probability that your lightbulb will work one year after purchase, given all the available lightbulbs in the store have been there for a year already? (Leave your answers in powers of e, e.g. $ae^b$ for some constants $a$ and $b$.)

   (b) Let random variable X be uniform in the interval $[0, 3]$. Show how to generate random variable $Y$ with pmf as defined below based on $X$.

$$p_Y(k) = \begin{cases} 0.5, & k = 0 \\ 0.4, & k = 1 \\ 0.1, & k = 2 \end{cases}$$

5. **[20 points]** The two parts of the problem are unrelated.

   Hint: The derivative of the arcsin($x$) function equals $\frac{1}{\sqrt{1-x^2}}$.

   (a) A real-valued continuous random variable $X$ is said to have an arcsin(-1,1) distribution if it has a CDF of the form

$$F_X(x) = c \arcsin\left(\sqrt{\frac{x+1}{2}}\right),$$

   where $x \in (-1, 1)$, and $c$ is some real-valued constant. Recall that arcsin($a$) stands for the inverse sin function, that is, a function that returns the angle (in radians) whose sin equals $a$.

   Determine the constant $c$ and describe the values of the CDF $F_X(x)$ outside of the interval $(-1, 1)$ that will make it into a valid CDF. Find the pdf $f_X(x)$ of $X$ and determine the values of the pdf for $x = -1$ and $x = 1$. Why are these values allowed?

(b) Let $U$ be a random variable uniformly distributed in $[-\pi, \pi]$. Find the CDF and pdf of the random variable $X = \sin(U)$.