University of Illinois Spring 2018

ECE 313: Hour Exam II

Wednesday, April 11, 2018
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section:
☐ C, 10:00 a.m. ☐ G, 11:00 a.m. (Alvarez) ☐ D, 11:00 a.m. (Dominguez-Garcia)
☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided. DO NOT convert answers to decimal fractions (for example, write $\frac{3}{4}$ instead of 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 22 points	
2. 18 points	
3. 22 points	
4. 18 points	
5. 20 points	
Total (100 points)	

1. [**22 points**] Emails arrive to Professor X's inbox according to a Poisson process with rate $\lambda = 0.2$ emails per minute. Measure time in minutes and consider a time interval beginning at time $t = 0$. Find the probability of the following events. (Leave your answers in powers of e, e.g. ae^b for some constants a and b.)

   (a) E_1: Three emails arrive within the first minute.

   (b) E_3: The time it takes for the first email to arrive is at least 5 minutes.

(c) E_3: The time it takes for the second email to arrive is at least 2 minutes.

2. **[18 points]** The random variable X has the $N(-6, 36)$ distribution. Express the answers to the following questions in terms of the Q function.

   (a) Obtain $P\{|X| > 6\}$.

   (b) Let $Y = aX + b$, where $P\{2 < Y\} = \frac{1}{2}$. Determine a valid pair of values of $a \neq 0$ and b.

3. **[22 points]** Suppose X and Y are independent random variables such that X is uniformly distributed over the interval $[0, 1]$ and Y is exponentially distributed with parameter $\lambda > 0$.

(a) Find the joint CDF $F_{X,Y}$ for all (u, v).

(b) Find $P(Y = X)$.

(c) Find $P(Y \leq 4X)$.

4. **[18 points]** The two parts of the problem are unrelated.

(a) The lifetimes of light bulbs (in years) produced by two companies, *Fos* and *Illuminati*, follow exponential distributions with parameters $\lambda = 1$ and $\lambda = 2$, respectively. You purchased a random lightbulb from a store that does not carry manufacturer labels, but carries the same number of products by *Fos* and *Illuminati*. What is the probability that your lightbulb will work one year after purchase, given all the available lightbulbs in the store have been there for a year already? (Leave your answers in powers of e, e.g. ae^b for some constants a and b.)

(b) Let random variable X be uniform in the interval $[0, 3]$. Show how to generate random variable Y with pmf as defined below based on X.

$$p_Y(k) = \begin{cases} 0.5, & k = 0 \\ 0.4, & k = 1 \\ 0.1, & k = 2 \end{cases}$$

5. **[20 points]** The two parts of the problem are unrelated.

Hint: The derivative of the arcsin(x) function equals $\frac{1}{\sqrt{1-x^2}}$.

(a) A real-valued continuous random variable X is said to have an arcsin(-1,1) distribution if it has a CDF of the form

$$F_X(x) = c \text{ arcsin}\left(\sqrt{\frac{x+1}{2}}\right),$$

where $x \in (-1, 1)$, and c is some real-valued constant. Recall that arcsin(a) stands for the inverse sin function, that is, a function that returns the angle (in radians) whose sin equals a.

Determine the constant c and describe the values of the CDF $F_X(x)$ outside of the interval $(-1, 1)$ that will make it into a valid CDF. Find the pdf $f_X(x)$ of X and determine the values of the pdf for $x = -1$ and $x = 1$. Why are these values allowed?

(b) Let U be a random variable uniformly distributed in $[-\pi, \pi]$. Find the CDF and pdf of the random variable $X = \sin(U)$.