University of Illinois Spring 2018

ECE 313: Hour Exam I

Wednesday, February 28, 2018
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ________________________________

NetID: ________________________________

Signature: ________________________________

Section:
☐ C, 10:00 a.m. ☐ G, 11:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **six** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 12 points ____________
2. 10 points ____________
3. 20 points ____________
4. 18 points ____________
5. 22 points ____________
6. 18 points ____________

Total (100 points) ____________

1. **[12 points]** A blood test gives readings of an indicator X according to the following likelihood matrix

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0.0	0.35	0.6	0.05
H_0	0.4	0.3	0.2	0.1

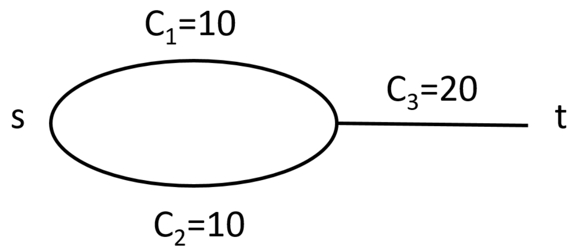
The priors are $(\pi_0, \pi_1) = (0.2, 0.8)$.

(a) What is the maximum likelihood (ML) decision rule? Compute p_{miss} for the ML decision rule.

(b) What is the maximum a posteriori (MAP) decision rule? Compute p_{e} for the MAP decision rule.

(c) What is the decision rule that minimizes p_{miss}?

2. **[10 points]** Consider the following network. Each link fails with probability p.



(a) What is the outage probability?

(b) What is the probability that the network has capacity 10?

3. **[20 points]** The three parts of this problem are unrelated.

(a) Consider a random variable Y. It is known that $E[-3Y-2] = 4$ and that $Var(-3Y-2) = 36$. Determine $E[Y]$ and $Var(Y)$.

(b) Consider rolling a fair die and flipping a fair coin. Define a random variable X which is equal to the number shown in the die if the coin shows heads and twice the number in the die if the coin shows tails. Obtain the pmf of X.

(c) Let Z be a random variable with pmf $p_Z(k) = \frac{c}{(2^k)}$ for $k \in \{1, 2, 3, 4\}$ and zero else. Determine the value of the constant c, of the mean $E[Z]$ and of $E[2^Z]$.

4. **[18 points]** If an intercontinental ballistic missile (ICBM) is launched from a nuclear submarine in a certain part of the world, a radar system might be able to detect its presence while airborne and launch a second missile to intercept the ICBM. Let M denote the event that an ICBM has been launched, and assume $P(M) = 0.001$. Let D denote the event that the radar system alerts of the presence of a missile, and assume $P(D|M) = 0.99$. In addition, assume that $P(D^c|M^c) = 0.99$.

   (a) Let E denote the event that the radar system makes a mistake. Compute $P(E)$.

   (b) Given that the radar system detects the presence of an ICBM, compute the probability that an ICBM has actually been launched.

5. **[22 points]** This problem considers two basketball teams A and B. The three parts are unrelated.

   (a) We estimate the probability that team A beats team B (denoted by p) by having them play against each other n times. If we want to estimate p within 0.1 and with 75% confidence (using the confidence interval based on the Chebychev bound), how many games they should play?

   (b) The expected number of points made by team A on a game is 75. Provide a lower bound on the probability that team A makes less than 90 points on a given game.

(c) We want to estimate the probability that team A beats team B. To that end, we make them play against each other until team A beats team B 3 times, which happens at the 10th game. What is the Maximum Likelihood (ML) estimation of the probability that team A beats team B? Assume each game is independent of previous games.

6. **[18 points]** The two parts are unrelated.

   (a) A basketball team is composed of 10 players, 6 of which are considered to be "very good", and 4 "average". At any given time on a basketball game, only 5 players are in the field. Let's assume that we decide on the 5 players at the beginning of the game, and that they play the whole game. Let's denote the selected 5 players as the *playing team*. We say that the *playing team* is good if at least 3 out of the 5 players are "very good". Find the probability that a *playing team* is good.

   (b) A committee of four people is to be selected from a group of four man and four women. What is the probability that the committee is not gender biased (i.e., it has the same number of men and women)?