ECE 313 (Section G)
Homework 9
Due Date: Wednesday, April 19, 11:00 AM in the class

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1 – Failure data of 10 electronic components is given below.

Failure number	Operating time (hr)
1	8
2	20
3	34
4	46
5	63
6	86
7	111
8	141
9	186
10	266

Compute the failure density $f_d(t)$ and the hazard rate $z_d(t)$ to complete the table in the following worksheet. *Make sure to **show your work** (equations) when filling in the worksheet. Two calculations are provided to guide you.*

Note: Remember that, if there are N components and $n(t)$ represents the number of survivors at time t_i, then the failure density is given by,

$$f_d(t) = \frac{[n(t_i)-n(t_i+\Delta t_i)]}{N \times \Delta t_i} \qquad \text{for } t_i < t \leq t_i + \Delta t_i$$

And the hazard rate is given by

$$z_d(t) = \frac{[n(t_i)-n(t_i+\Delta t_i)]}{n(t_i) \times \Delta t_i} \qquad \text{for } t_i < t \leq t_i + \Delta t_i$$

Time interval (hrs)	Failure density $(f_d(t))$ $(\times 10^{-2})$	Hazard rate $(z_d(t))$ $(\times 10^{-2})$
0-8	$\dfrac{1}{10\times 8} = 1.25$	
8-20		$\dfrac{1}{9\times 12} = 0.93$
20-34		
34-46		
46-63		
63-86		
86-111		
111-141		
141-186		
186-266		

Problem 2 – Let the joint pdf of X and Y be given by:

$$f(x,y) = e^{-\frac{x}{\alpha}}ye^{-y^2}, for\ x > 0, y > 0$$

where $\alpha \neq 0$. The random variables X and Y are said to have a two-dimensional (or bivariate) normal pdf.

a) Show that the marginal pdf's of X and Y are:

$$f(x) = \frac{1}{2}e^{-\frac{x}{\alpha}} \quad and \quad f(y) = \alpha y e^{-y^2}$$

b) Find the values of α, for which X and Y are independent.

Problem 3 – Let X_1 and X_2 be two independent random variables exponentially distributed with parameters λ_1 and λ_2.

   a) Find the pdf of $Z = \min(X_1, X_2)$.
   b) Find the pdf of $R = \dfrac{X_1}{X_2}$

Problem 4 – Suppose that random variables X and Y are jointly distributed with joint density $f_{X,Y}(x, y) = c(1 + xy)$ for $2 \leq x \leq 3$ and $1 \leq y \leq 2$. $f_{X,Y}(x, y) = 0$ otherwise.
   a) Find the constant c.
   b) Find the marginal pdf's of X and Y.

Problem 5 – Let X be a random variable exponentially distributed with parameter λ. Find the CDF of
   a) $Y = \exp(X)$
   b) $Z = \min(X, 3)$