**ECE 313 (Section G)**

**Homework 5**

**No Due Date**

**Problem 1 –** Suppose two fair dice are rolled. Let X be the minimum of the two numbers showing. For example, if a 2 shows on the first die and a 5 shows on the second, then X = 2. That is, X((2, 5)) = 2. In general, X((i, j)) = min{i, j} for (i, j) ∈ Ω. Find the numerical values of the following quantities (show your work to get credit):

1. Calculate and draw the pmf of X.
2. Draw the cumulative distribution function (CDF) of X.
3. P{X ≤ 1}
4. P{X < 1}
5. P{X ≥ 6}
6. P{X = 6}
7. P{X > 6}
8. P{3 ≤ X ≤ 6}
9. P{|X − 3| ≤ 0.1}.

**Problem 2 –** From previous inspections, in every 100 memory chips produced by a plant 8 are known to be defective. A sample of 10 memory chips is drawn randomly from each week’s production. If X is the number of defective chips found in the weekly samples, find:

1. The pmf of X. What is the distribution of X?
2. The probability that none of the chips is defective.
3. The probability that exactly one chip is defective.
4. The probability that more than 2 chips are defective.
5. The probability that more than 2, but fewer than 6 chips are defective.

**Problem 3 –** There are 100 boxes and only one of them contains a gift. A robot randomly selects boxes until it finds the one with the gift. Let *N* be the number of attempts required to find the gift (this includes the attempt in which the box containing the gift was selected). Determine the probability mass function (pmf) of *N,* upto 5 values:

1. If the previously selected boxes are excluded in the next attempt (sampling without replacement).
2. If the previously selected boxes are included in the next attempt (sampling with replacement).
3. Assuming that the previously selected boxes are included in the next attempt, calculate the probability that
   1. The gift will be found on the 5th attempt.
   2. The gift will be found on the 15th attempt, given that gift wasn’t found in the first 10 attempts.
4. What can you deduce from the answers you got for part c)?

**Problem 4 –** Consider the following program segments consisting of a *while* loop:

*int B = randInt();*

*// randInt() is a random integer generator that returns an integer between* ***1 and 100****.*

*// All integers are equally likely to be generated.*

1. *while (B ≥ 65){*

*execute S;*

*B = randInt();*

*}*

1. *do{*

*execute S;*

*B = randInt();*

*}while (B ≥ 65);*

In the first program segment, the loop is taken while the Boolean expression (*B ≥ 65*) is TRUE. Variable B gets a new random value after each execution of S. In the second program segment, the condition is tested after the execution of the loop.

If the successive executions of the loop are independent from each other, then let *X* be the number of times the body (or the statement-group S) of the loop is executed.

1. Let *p* be the probability of the loop condition (B ≥ 65) being **TRUE**. Find the value of *p*.

For each of the program segments **i** and **ii**,

1. Determine the set of values that the random variable *X* might take.
2. Derive the probability mass function (pmf) of *X* and determine the type of distribution, **in terms of p**.

**Problem 5 –** A company manufactures CD-ROMs with a mean diameter of 15cm and a standard deviation of 5mm. We can approximate the distribution of the diameters by a normal distribution. After manufacturing, the CDs which have a diameter between 14cm and 16cm are packed for sales; the rest are discarded. What percent of the CDs  
  
a) will have a diameter greater than 15.5cm?   
  
b) will be discarded?

(You can use the standard normal CDF table provided in the next page)

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**Problem 6.** Suppose 8% of the rocket launch at ACME spacecraft company fail. Our goal is to illustrate the use of the Poisson approximation for the Binomial.

Calculate the probability of having exactly one failed launch P(X=1) from a sample of 20 launches:

1. Using Binomial formula
2. Using Poisson formula

Compare the two results and explain your answer