ECE 313 (Section G)
Homework 1

Due Date: Wednesday, Feb 1st , 11:00 AM in the class

Write your name and NetID on top of all the pages. Show your work to get partial credit.

Problem 1 - Describe a possible sample space and determine its size for each of the following experiments:

a) The following for loop is executed in a program. *p* is a variable that can be either TRUE or FALSE each time through the loop, A and B are statements that can be executed. In how many ways, can **exactly** 4 'A's occur?

```
for (int i = 0; i < 13;  i++)
{
     if (p == TRUE)
          A;
     else
          B;
}
```

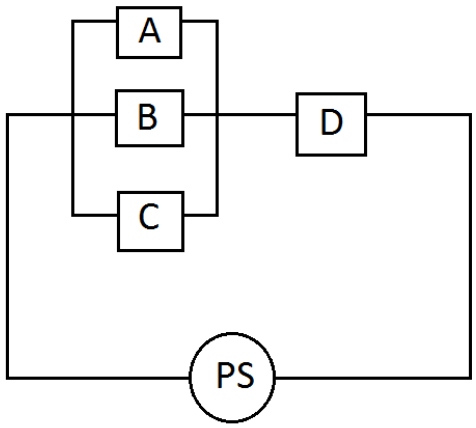
b) We pick a random number from the set of odd numbers with distinct digits between 100 and 1000.

c) We pick a random positive integer less than 4,000 that has exactly one 8.

d) Three students stop by at a library in sequence and each plan to borrow four books to read. There are *n* distinct books ($n \geq 12$) available at the library (after a book is taken by the first student, it becomes unavailable to the second and third student). We are interested in the number of possible ways that twelve books could be borrowed.

Problem 2 – Most cyber security applications use some form of preemptive attack detection. By detecting an attack before its completion, the damage could be prevented or minimized. The success of preemptive attack detection depends on tagging a set of past events as 'malicious' or 'benign'. In a set of 10 past events, what is the probability that there were **exactly** 4 consecutive 'malicious' events?

Problem 3 – Below diagram shows a simplified industrial control system. PS is a power source and components A, B, C, and D are independent elements representing various functionalities and controls. Each component can fail independently and when a component fails, the path becomes blocked. However, for this control system to be active, there should be at least one complete cycle in this diagram.
For example, PS->A->D->PS is a complete cycle.



a) What is a suitable sample space S for this experiment? List all elements in S.

b) Identify the event E corresponding to the statement "the control system is active"

c) If each component fails with a probability *p*, what is probability of event E (i.e. P(E))?

Problem 4 – A chef wants to make a fruit salad and has contains 9 bananas, 5 apples, 3 peaches and 4 oranges in a bag. Four pieces of fruits are selected randomly from the bag. Find the probability of the following events:

a) 1 piece of each fruit.

b) 2 apples and 2 peaches.

c) At least one apple and at least one orange.

d) At most 2 bananas

Problem 5 – An experiment consists of observing the contents of a 16-bit register. We assume that all 65536 byte values are equally likely to be observed.

a) Let A denote the event that the Least Significant Bit (LSB) is a ZERO. What is P(A)?

b) Let B denote the event that the register contains 5 ONEs and 11 ZEROs. What is P(B)?

c) What is the probability of at least one of A or B occurs, i.e. what is P (A∪ B)?

d) What is the probability that exactly one of A and B occur, i.e. what is P (A ⊕ B)?

e) What is the probability of at least one of A or B does not occur?

Problem 6 – Here we consider a simple game called the Prisoner's Dilemma. Two members of a criminal gang, ***A*** and ***B***, are arrested and are under investigation. Police don't have enough evidence, which makes them long for the prisoner's confession. Criminals have a choice to confess (C) or to deny (D). To encourage the criminals to confess, police offers a bargain:

- If ***A*** and ***B*** both confess, each of them serves 2 years in prison
- If ***A*** betrays ***B*** (***A*** confesses, ***B*** denies), then ***A*** will be set free while ***B*** will serve 3 years in prison (and vice versa)
- If ***A*** and ***B*** both remain silent, both of them serve 1 year in prison

a) What is the sample space in the Prisoner's Dilemma?

A decides to confess and betray ***B***. ***B*** is reluctant to betray ***A***. Therefore, B is thinking of confessing with a probability of 0.4. (***B*** does not know what ***A*** will choose and vice-versa).

b) What is the possibility of ***A*** being set free, and ***B*** serving 3 years in prison?

c) What is the possibility of both ***A*** and ***B*** serving 2 years in prison?