**ECE 313 (Section G)**

**In-Class Project 6 Solution - Wednesday, Apr 19**

Write your name and NetID here:

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Two biomedical signals, the blood pressure (ABP) and the heart rate (HR), are measured to detect the abnormalities of a patient in an intensive care unit (ICU). Assume that the blood pressure sensor outputs a value *X* and the heart rate sensor outputs a value *Y*. Both *X* and *Y* outputs have possible values of {0, 1, 2}, representing different ranges of ABP and HR values, with larger numbers tending to indicate that a patient abnormality is present.

Let *H0* be the hypothesis there is no abnormality, and *H1* be the hypothesis an abnormality is present. The likelihood matrices for *X* and for *Y* are shown:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **X=0** | **X=1** | **X=2** |
| $$H\_{1}$$ | 0.2 | 0.3 | 0.5 |
| $$ H\_{0}$$ | 0.7 | 0.1 | 0.2 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Y=0** | **Y=1** | **Y=2** |
| $$H\_{1}$$ | 0.2 | 0.0 | 0.8 |
| $$H\_{0}$$ | 0.7 | 0.2 | 0.1 |

Suppose, given one of the hypotheses is true, the sensors provide independent readings conditioned on the same hypothesis, so that:

1. Find the likelihood matrix for the observation (*X*, *Y*), and indicate the ML decision rule. To be definite, break ties in favor of *H1*.
2. Find and for the ML rule found in part (a).
3. Suppose, based on past experience, prior probabilities are assigned as: . Compute the joint probability matrix and indicate the MAP decision rule.
4. For the MAP decision rule, compute,, and the probability of error .

( )

1. Using the same priors as in part (c), compute the unconditional error probability for the ML rule from part (a). Is it smaller or larger than the found for the MAP rule in part (d)?
2. **(30 points; 20 points for filling out the table, 10 points for marking the correct decision.)** The likelihood matrix for observation *(X, Y)* is the following:$Type equation here.$

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X,Y** | **0,0** | **0,1** | **0,2** | **1,0** | **1,1** | **1,2** | **2,0** | **2,1** | **2,2** |
| $$H\_{1}$$ | 0.04 | 0 | **0.16** | 0.06 | 0 | **0.24** | 0.10 | 0 | **0.40** |
| $$H\_{0}$$ | **0.49** | **0.14** | 0.07 | **0.07** | **0.02** | 0.01 | **0.14** | **0.04** | 0.02 |

The ML decisions are indicated by the bold elements. The larger number in each column is bold. Note that the row sums are both one.

1. **(15 points; 8 points for p\_false-alarm, 7 points for p\_miss.)** For the ML rule, $p\_{false-alarm}$ is the sum of the entries in the row for H0 in the likelihood matrix that are not bold. So $p\_{false-alarm}$ = (0.07 + 0.01 + 0.02 ) = 0.1

(assuming an uniform prior where H\_0 and H\_1 are equaly likely)

For the ML rule, $p\_{miss}$ is the sum of the entries in the row for H1 in the likelihood matrix that are not bold. So $p\_{miss}$ = (0.04 + 0.06 + 0.10 ) = 0.2

1. **(30 points; 20 points for filling out the table, 10 points for marking the correct decision.)** The joint probability matrix is given by:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X,Y** | **0,0** | **0,1** | **0,2** | **1,0** | **1,1** | **1,2** | **2,0** | **2,1** | **2,2** |
| $$H\_{1}$$ | 0.008 | 0 | 0.032 | 0.012 | 0 | **0.048** | 0.02 | 0 | **0.08** |
| $$H\_{0}$$ | **0.392** | **0.112** | **0.056** | **0.056** | **0.016** | 0.008 | **0.112** | **0.032** | 0.016 |

(The matrix specifies *P(X = I, Y = j | Hk)* for each hypothesis *Hk* and for each possible observation value *(i, j)*. The 18 numbers in the matrix sum to one. The MAP decisions are indicated by the bold elements in the joint probability matrix. The larger number in each column is bold.)

1. **(15 points; 6 points for p\_false-alarm, 6 points for p\_miss, 3 points for p\_e. Give half the credit if there is no division by the prior)**

For the MAP rule,

P(false alarm) = (0.008 + 0.016) / 0.8 = 0.024 / 0.8 = 0.03

and:

P(miss) = (0.008 + 0.032 + 0.012 + 0.02) / 0.2 = 0.072 / 0.2 = 0.36

Thus, for the MAP rule, = 0.8\*0.03 + 0.36\*0.2 = 0.096

1. **(10 points; 5 points p\_e of ML rule, 5 points for comment)**

Using the conditional probabilities found in (a) and the given values of and yields that for the ML rule: $p\_{e}$ = 0.8\*(0.1)+0.2\*(0.2) = 0.12; which is larger than the value 0.096 for the MAP rule, as expected because of the optimality of the MAP rule for the given priors.

**Note:** student may assume an equally likely prior for ML rule, so p\_e = 0.5\*0.1 + 0.5\*0.2 = 0.15, and that answer is acceptable. P\_e = 0.1 + 0.2 = 0.3 is also accepted as error for ML rule.