Announcements

- HW0 is due today
- HW1 is released and will be due on Wed, Feb 1st
- Class activity
  - Group of 3 to 4
  - Each person in the group has to write their own answer
  - Extra sheets are available
  - Remember to write your name and netid at the top of the sheet

Concepts overview

Probability Axioms

- Summarizing:
  - A1 For any event A $P(A) \geq 0$
  - A2 $P(S) = 1$ (Prob of a certain event)
  - A3 $P(A \cup B) = P(A) + P(B)$
    A & B are mutually exclusive

    $A \cap B = \emptyset$

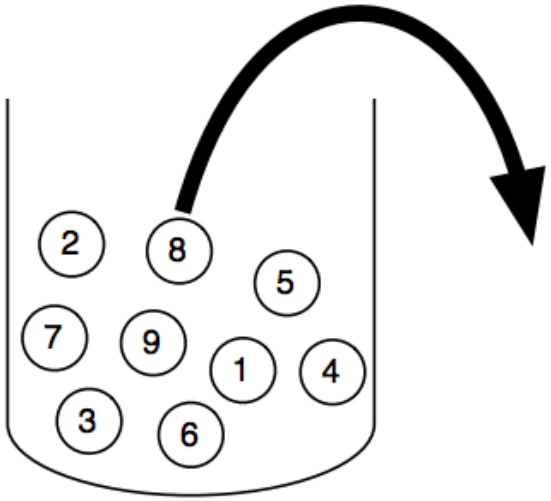
- Next:
  - Basic steps in problem solving
  - Combinatorial methods

Combinatorial Problems

- We are often concerned with selecting some number of objects from a total number of objects
- Sample Space consisting of a finite number (n) of points (elements, sample points, and outcomes)

- The sampling table gives the number of possible samples of size k out of a population of size n, under various assumptions about how the sample is collected.

Sampling table

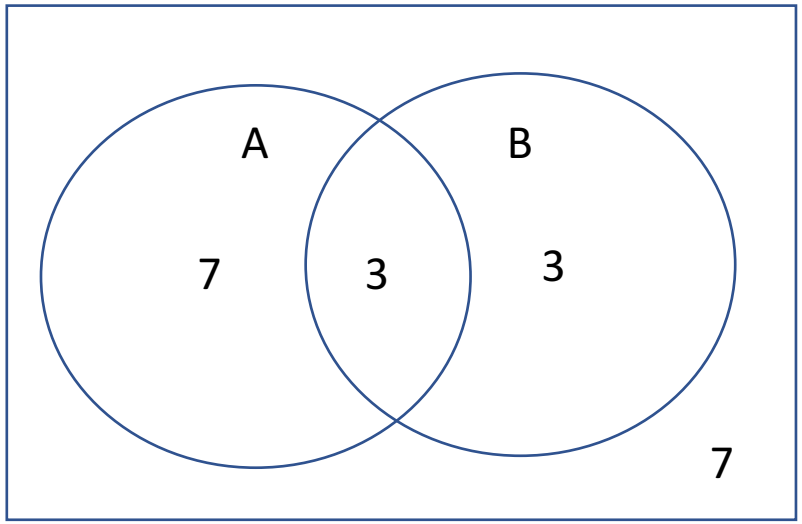


	Order Matters	Not Matter
With Replacement	n^k	$\binom{n+k-1}{k}$
Without Replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Adapted from Probability Cheatsheet v2.0, WZChen

Karnaugh map

- A: numbers which are multiples of two
- B: numbers which are multiples of three
- Universal set: numbers in the range 1 - 20



Venn diagram

	A	A^c	
B	3	3	6
B^c	7	7	14
	10	10	20

Karnaugh map

Basic Steps to Solving Problems

- Identify the sample space S
  - The sample space S must be chosen so that all its elements are mutually exclusive and collectively exhaustive, I.e., no two elements can occur simultaneously and one element must occur on any trial.
- Assign probabilities to the elements in S
  - This assumption must be consistent with the axioms A1 through A3
- Identify the events of interests
  - The events are described by statements and need to be recast as subsets of the sample space
- Compute desired probabilities
  - Calculate the probabilities of the events of interest using axioms and any derived laws
- Develop the Insight about the system/experiment

Class Activity 1

Consider a bag with one set of all alphabets, i.e., a to z in lowercase. Letters are picked from the set without replacement

1. What is the probability that the first two letters are vowels?
2. What is the probability that the first two letters are a combination of vowel and consonant?
3. What is the probability that the first three letters form a palindrome?

How would your answers change if the bag contains two sets of alphabets?

Solution

- There are 26 alphabets present with 5 vowels and 26 consonants
- Probability of first two letters being vowels = 5C2/26C2
- Probability of first two letters being a vowel and a consonant = probability of vowel first and a consonant second + probability of a consonant first and a vowel second = (2)*(5/26)*(21/25)
- At least two same alphabets are needed to make a palindrome. For one set of alphabets, no palindrome can be formed => probability of first three letters forming a palindrome is 0

Solution

When two sets of alphabets are present:

- Probability of first two letters being vowels = 10C2/52C2
- Probability of first two being a vowel and a consonant = (10/52) * (42/51) * (2)
- Probability of palindrome = 52/52 * 50/51 * 1/50

Class activity 2. Programming language assignment

Of 40 students in a class,

- 12 do not know Python
- 15 know both Python and Matlab
- 5/9 of students who do not know Matlab, use Windows
- 8 students use Windows and know both Python and Matlab
- 7 students do not know Matlab and do not use Windows, but know Python
- 2/3 of the students do not know Python and do not use Windows, know Matlab

Class activity 2. Programming language assignment

Let’s pick two students (A and B) from 40 students and assign them to a group to do programming assignments. Compute probabilities that

1. Both students know Python and Matlab
2. Both students know Python
3. Both students do not know either Matlab or Python
4. Student A knows a language that Student B does not know
5. Both students use Windows
6. Both students use Windows and know Python

Show your work. (Use a Karnaugh map. Fill in numbers or variables, trying to minimize the number of variables and equations needed.)

Solution

- There are 40 students in the class, and we have:
- 12 do not know Python
- => $|P| = 40 - 12 = 28$
- 15 know both Python and Matlab
- => $|P \cap M| = 15$
- 5/9 of students who do not know Matlab, use Windows
- => $|M^C \cap W| / |M^C| = 5/9$
- 8 students use Windows and are know both Python and Matlab
- => $|W \cap P \cap W| = 8$
- 7 students do not know Matlab and do not use Windows, know Python
- => $|M^C \cap W^C \cap P| = 7$
- 2/3 of the students who (do not know Python, do not use Windows, know Matlab
- => $|P^C \cap W^C \cap M| / | P^C \cap W^C | = 2/3$

Solution

- We fill in the values in the following Karnaugh map:

	Python, Matlab	Python, Matlab^C	Python^C, Matlab^C	Python^C, Matlab
Windows	8	28 - (8 + 7 + 7) = 6	12 - x - y - z	x
Windows^C	15 – 8 = 7	7	y	z

- These equations must hold
  - (6 + (12 – x – y – z)) / 6 + 7 + (12 – x –y – z) + y = 5/9
    - 18-x-y-z/25-x-z=5/9
  - z / (y + z) = 2/3

Solution

- We fill in the values in the following Karnaugh map:

	Python, Matlab	Python, Matlab^C	Python^C, Matlab^C	Python^C, Matlab
Windows	8	6	4	5
Windows^C	7	7	1	2

- 18-x-y-z/25-x-z=5/9
  - 17y + 4x = 37
- z / (y + z) = 2/3 => z = 2y

x = 5, y = 1, z = 2

Solution

- We fill in the values in the following Karnaugh map:

	Python, Matlab	Python, $Matlab^C$	$Python^C$, $Matlab^C$	$Python^C$, Matlab
Windows	8	6	4	5
$Windows^C$	7	7	1	2

- Number of students who know Python: 8 + 6 + 7 + 7 = 28
- Number of students who know Matlab: 8 + 7 + 5 + 2 = 22
- Number of students who know both Python and Matlab: 8 + 7 = 15
- Number of students who know Matlab but do not know Python, and know Python but do not know Matlab: (5+2)+(6+7)=20
- Number of students who both do not know any language: 4 + 1
- Number of students who both use Windows: 8 + 6 + 4 + 5 = 23
- Number of students who both use Windows and use Python: 8 + 6 = 14

Solution

Let’s pick two students (A and B) from 40 students and assign them to a group for assignments. Compute probabilities that

- Both students know Python and Matlab: 15/40*14/39
- Both students know Python: 28/40*27/39
- Both students do not know (either Matlab or Python): 5/40*4/39
  - Note: some students interpret this questions as three possibilities: 1) both student do not know Matlab; 2) both students do not know Python; and 3) both students do not know (Python and Matlab). In this cases, students still get credit for their answer.
- Student A knows a language that Student B does not know. There are three possibilities.
  - A knows Python, B does not know Python
  - 28/40*12/39
  - A knows Matlab, B does not know Matlab
  - 22/40*18/39
  - A knows both Python and Matlab, B does not know both Python and Matlab
  - 15/40*5/39
  - The answer is: 28/40*12/39 + 22/40*18/39 - 15/40*5/39 (to avoid double counting)
- Both students use Windows: 23/40*22/39
- Both students use Windows and know Python: 14/40*13/39