Group activity: Conditional Probability Reliability Evaluation Applications

ECE 313

Probability with Engineering Applications

Lecture 5

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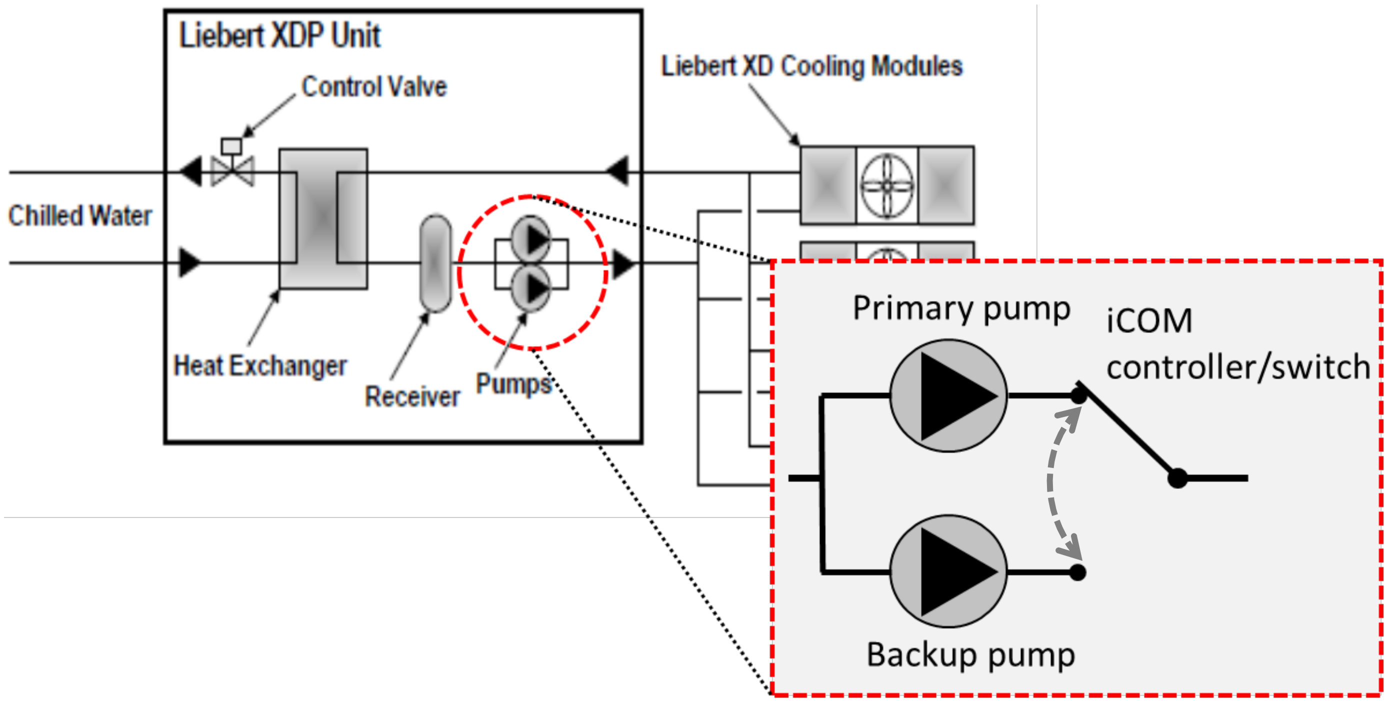
Today's Topics

- **In-class Group Activity**
- **Reliability Evaluation Applications**
  - series-parallel / non-series parallel systems

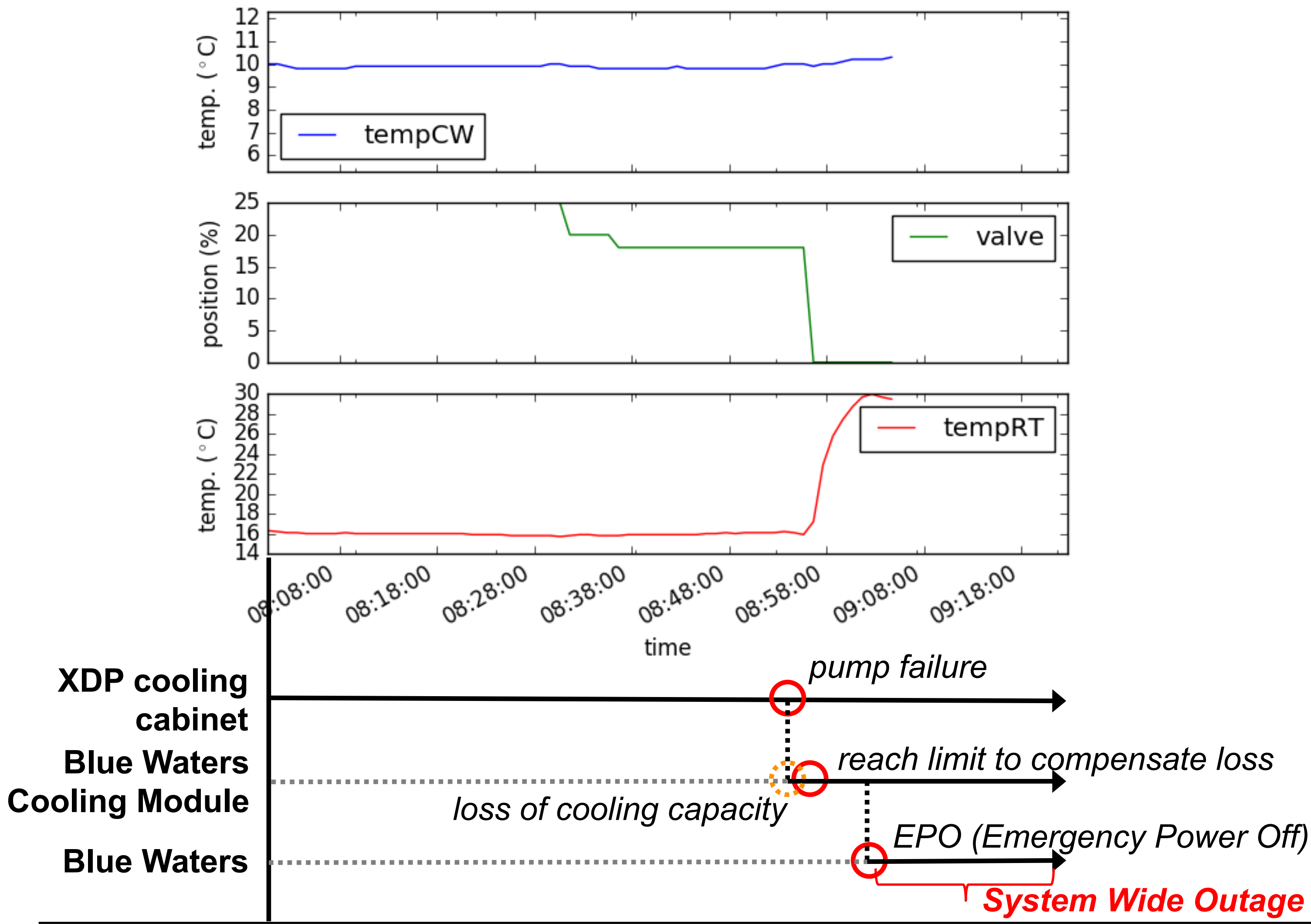
Cabinet cooling system



Cabinet cooling system

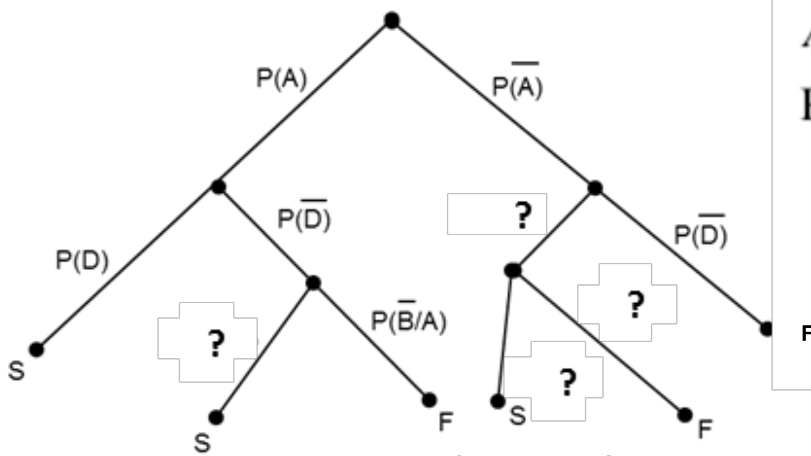
- A supercomputer needs chilled water cooling to keep the system operating within an acceptable temperature range.
- As shown in the previous slide, primary and backup pumps are used to maintain the flow of chilled water.
- An iCOM controller/switch monitors the status of the pumps and switches from primary to backup upon detecting a pump failure.
- Define the events:
  - A = "Primary pump functions correctly."
  - $\bar{A}$ = "Primary pump fails to function correctly."
  - B = "Backup pump functions correctly."
  - $\bar{B}$ = "Backup pump fails to function correctly."
  - D = "iCOM detects pump failure correctly."
  - $\bar{D}$ = "iCOM fails to detect pump failure or switches to the backup pump while the primary pump is operational."
  - F = "the pump system fails."
  - S = "the pump system is operational."
- Assume that event pairs *A* and *D* as well as *B* and *D* are independent but events *A* and *B* are dependent.

Blue Waters SWO due to Pump Failure



Cabinet cooling system (Question)

a) Complete the following tree by replacing ?'s with probability expressions.
S stands for the pump system is operational and *F* represents the pump system failure. Each path from the root to a leaf in the tree represents one of the ways that system would fail or succeed.



A and B are not independent :

$$P(A \mid B) \neq P(A)$$

b) Derive an expression for the failure probability of the pumping system highlighted in Figure 1.

$$P(F) = ?$$

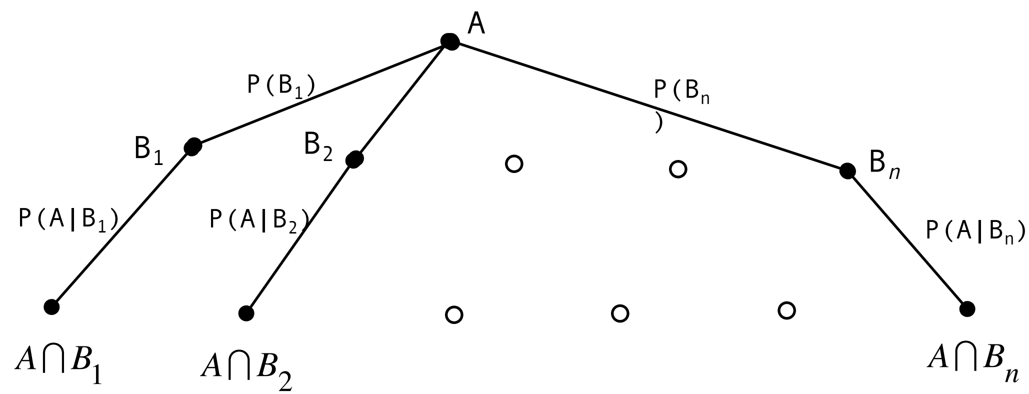
Cabinet cooling system (Question Cont.)

c) **Failure diagnosis of the system.** Derive an expression to find the probability of primary pumps fails, given that a failure has occurred.
Hint: use Bayes theorem and the law of total probability

Remember: Theorem of Total Probability

- This relation can be generalized with respect to the event space S' ={B_1,B_2,…,B_n) where B_1,B_2,…B_n are collectively exhaustive and mutually exclusive:

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$$



The Theorem of Total Probability

- The product of all probabilities from the root of the tree to any node equals the probability of the event represented by that node. P(A) can be computed by summing probabilities associated with all the leaf nodes of the tree.