University of Illinois Spring 2016

ECE 313: Exam II

Wednesday, April 13, 2016
7:00 p.m. — 8:15 p.m.
Aa-Hs in DCL 1320,
Ht-Lf in ECEB 1013,
Lg-Np in ECEB 1015,
Nq-Zz in ECEB 1002

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section:
☐ E, 9:00 a.m. ☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheets of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of 6 problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

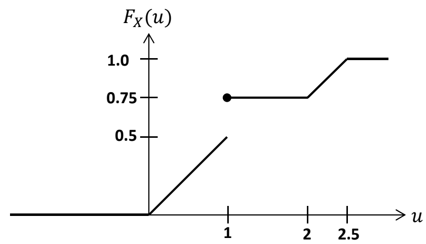
SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 10 points ______________
2. 20 points ______________
3. 18 points ______________
4. 18 points ______________
5. 18 points ______________
6. 16 points ______________

Total (100 points) ______________

1. **[10 points]** Consider the CDF in the figure below.



Carefully provide numerical answers for the following; partial credit will not be assigned for incorrect answers.

(a) Obtain $P\{0.5 < X \leq 1\}$.

(b) Obtain $P\{0.5 \leq X < 1\}$.

2. **[20 points]** An amplifier has gain $A = gR$ where g, the transconductance, is a positive constant, and R, the load resistance, is uniformly distributed in the interval $[9, 11]$.

   (a) Determine $E[A]$ and $\mathrm{Var}(A)$ in terms of g.

   (b) Determine the minimum value of g that would ensure that $P\{A \geq 10\} \geq 0.8$, i.e., at least 80% of the amplifiers have a gain of 10 or more.

3. **[18 points]** The three parts of this problem are unrelated.

(a) Let X be Gaussian with mean 1 and variance 4. Express $P\{(X-1)^2 < 16\}$ in terms of the Φ function.

(b) Let $X \sim \textit{Binomial}(160, 1/4)$. Use the Gaussian approximation with continuity correction to approximate $P\{45 \le X \le 50\}$ in terms of the Φ function.

(c) Let X have pdf $f_X(u) = \frac{u^2}{3}$ for $u \in (-2, 1)$ and zero else. Let $Y = -2X$ and obtain the pdf of Y, $f_Y(v)$ for all v.

4. **[18 points]** Suppose the times of tweets sent out by a certain celebrity form a Poisson process, with mean rate λ tweets per day.

   (a) What is the probability no tweets are sent in a two day period? (Hint: Your answer should depend on λ.)

   (b) How many days would it take, on average, for 20 tweets to be sent? (Hint: Your answer should depend on λ.)

   (c) Suppose the first day of observation there are 5 tweets, the second day 1 tweet, and the third day 2 tweets. What is $\widehat{\lambda}_{ML}$, the value of the maximum likelihood estimate of λ?

5. [**18 points**] Suppose under hypothesis H_1, the observation X has the $N(3,4)$ distribution. Under hypothesis H_0, the observation X has the $N(1,4)$ distribution.

   (a) Find the MAP decision rule in case the prior probabilities satisfy $\frac{\pi_0}{\pi_1} = e$ (where e is the base of natural logarithm, $e = 2.7182 \cdots$.)

   (b) Find a decision rule such that $p_{false\ alarm} = p_{miss}$.

6. **[16 points]** The following two parts are unrelated.

(a) Suppose X and Y have joint pdf $f_{X,Y}(u,v) = \begin{cases} Cuv & u^2+v^2 \leq 1, u \geq 0, v \geq 0 \\ 0 & \text{else,} \end{cases}$, for some appropriate constant C. Find the conditional pdf of Y given X, $f_{Y|X}(v|u)$, *and also determine whether* X *and* Y *are independent.* (Hints: It is not necessary to determine C. Be sure to specify what values of u $f_{Y|X}(v|u)$ is defined for, and specify for all $v \in \mathbb{R}$.)

(b) Suppose X and Y have joint pdf $f_{X,Y}(u,v) = \begin{cases} 0.5 & u \in [0,2], v \in [0,1] \\ 0 & \text{else.} \end{cases}$. Find the conditional pdf of Y given X, $f_{Y|X}(v|u)$, *and also determine whether* X *and* Y *are independent.*